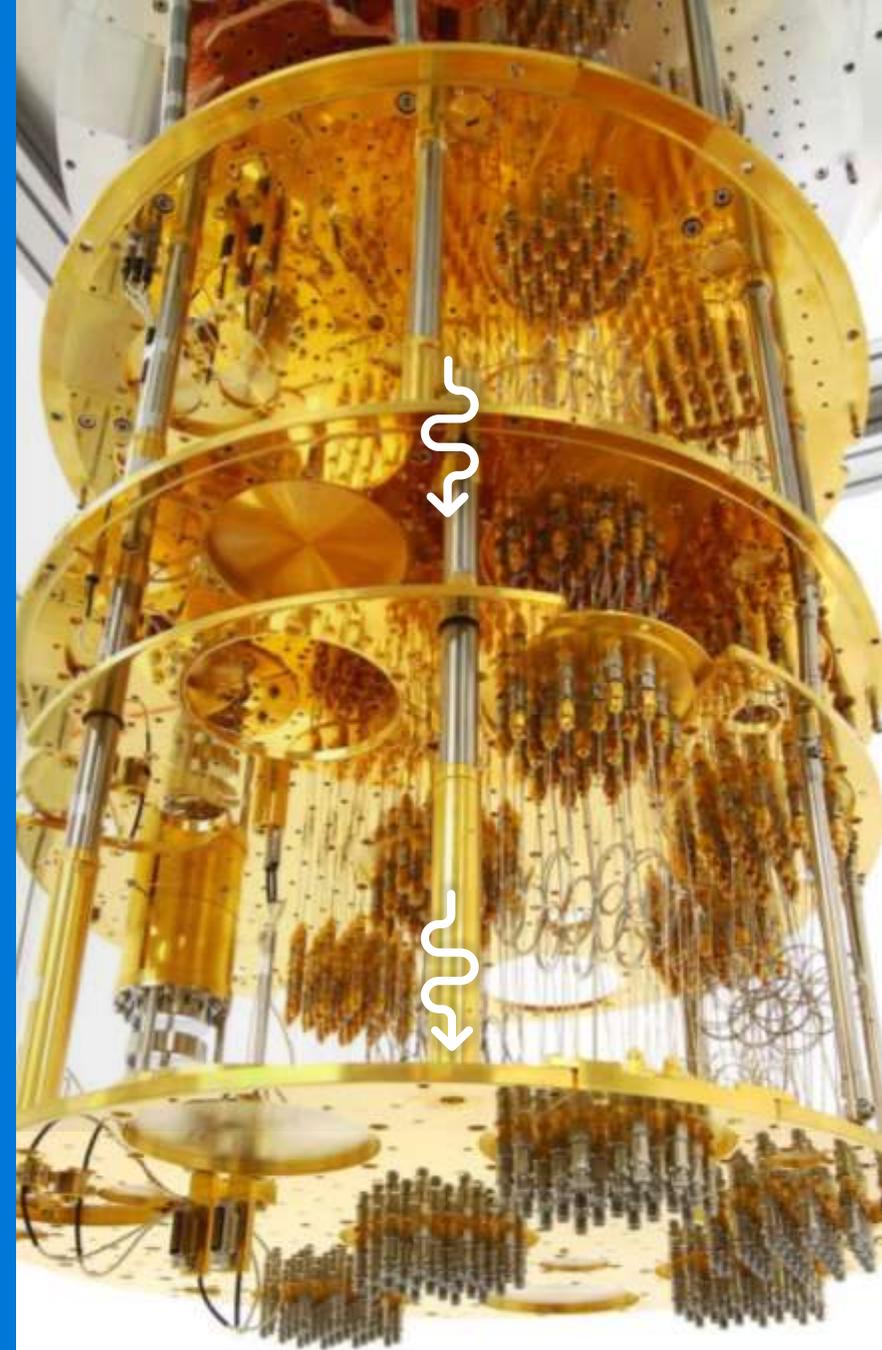




High Performance Quantum Computing

Prof. Matthias Troyer
Principal Researcher



Cray X-MP/28 at ETH Zurich in 1988



Beyond exascale computing



Enabling technologies for beyond exascale computing

- We are not referring to 10^{21} flops
- “Beyond exascale” systems as we are defining them will be based on new technologies that will finally result in the much anticipated (but unknown) phase change to truly new paradigms/methodologies.



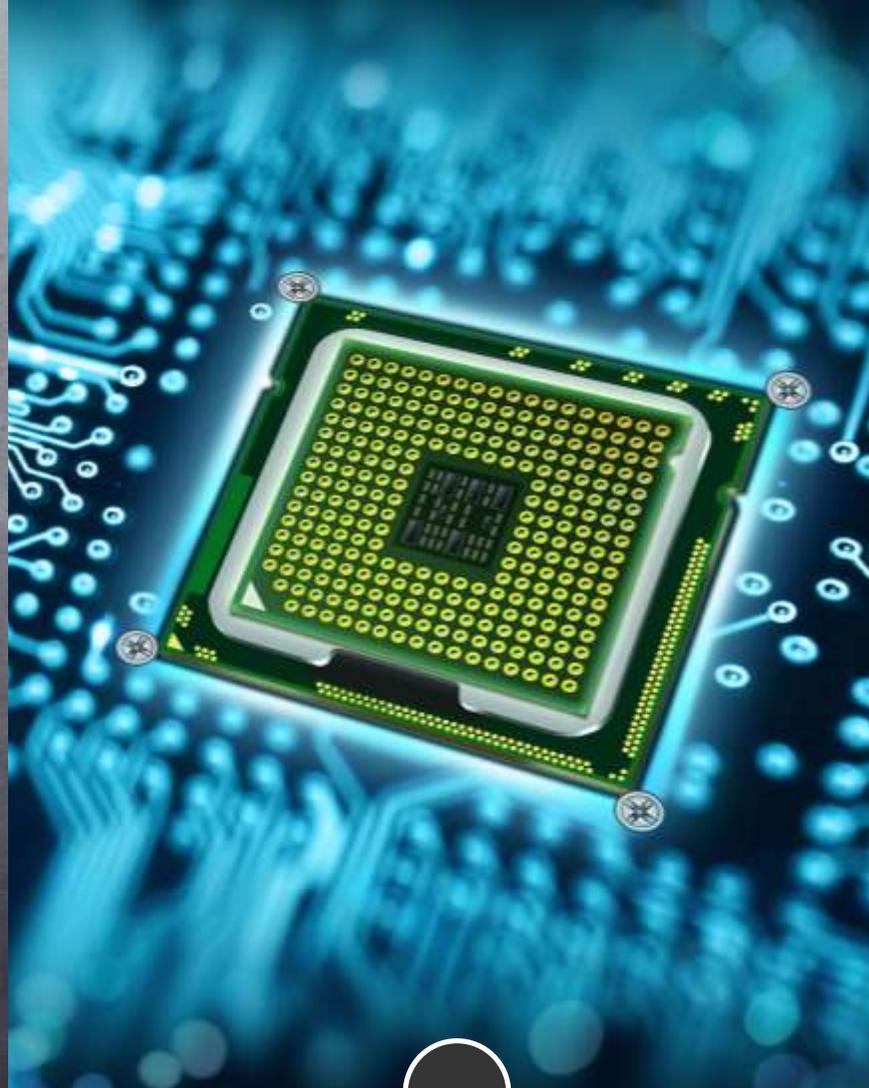
Paul Messina

*Director of Science
Argonne Leadership Computing Facility
Argonne National Laboratory*

*July 9, 2014
Cetraro*



2500
BC



20th
Century



21st
Century

Addressing classically intractable problems

Classical

1 billion
years

Quantum

100
seconds

Impact on Cryptography

Quantum computers break widely used public key encryption

RSA-2048 with 4100 qubits

ECC: Bitcoin with 2330 qubits



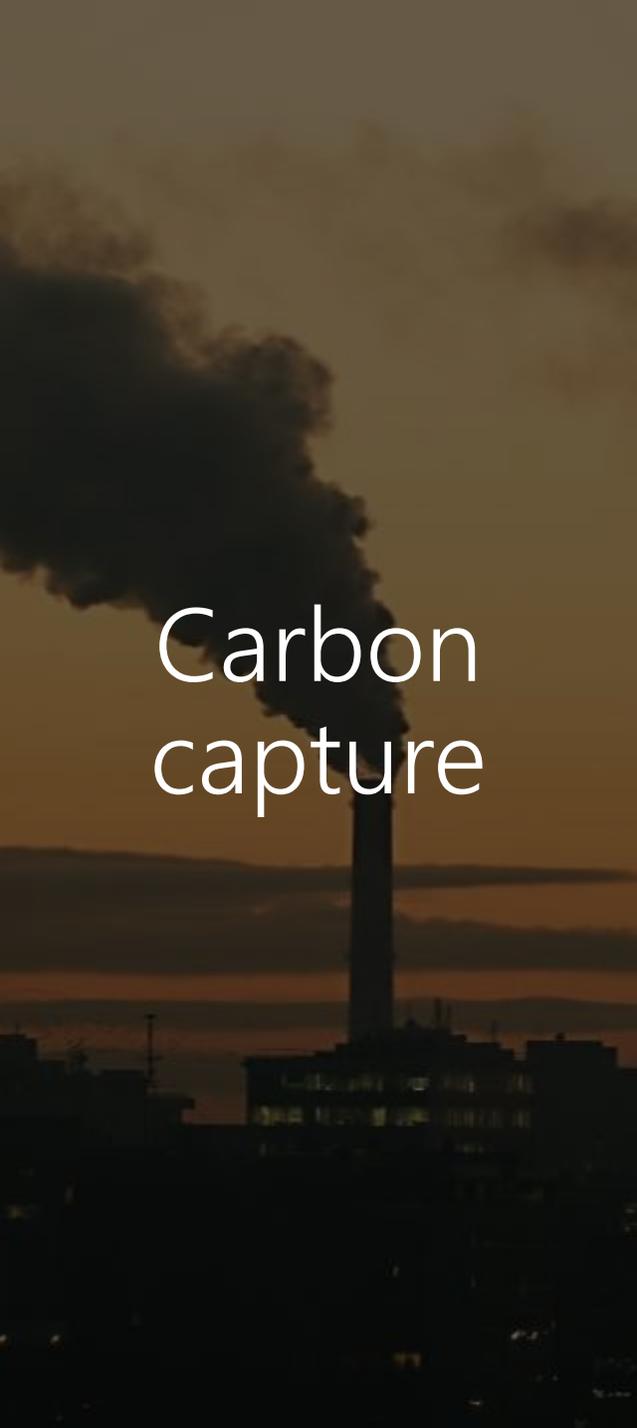
New quantum-safe cryptography

Quantum key distribution

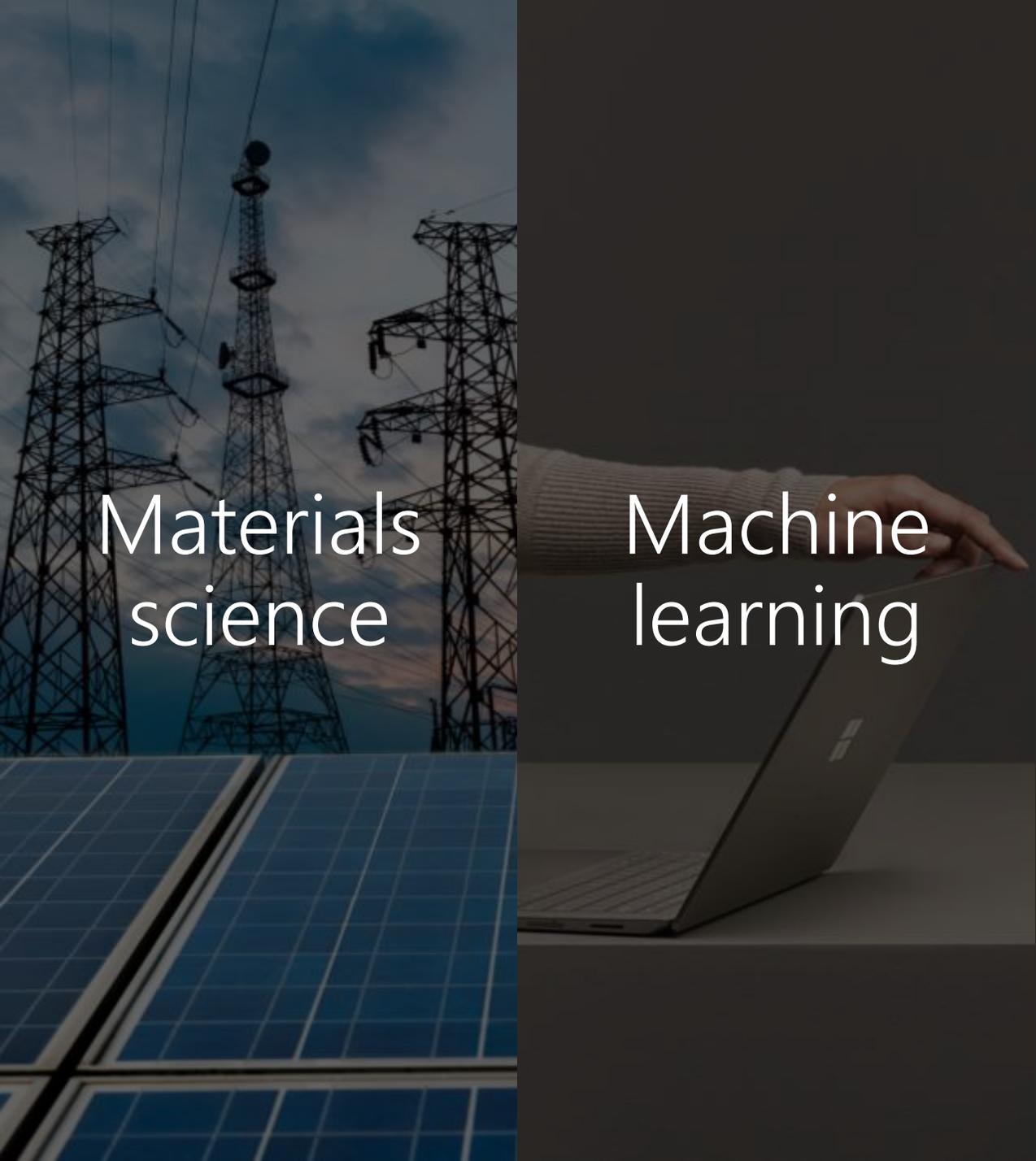
"Post-quantum" classical cryptography research at Microsoft



Nitrogen
fixation



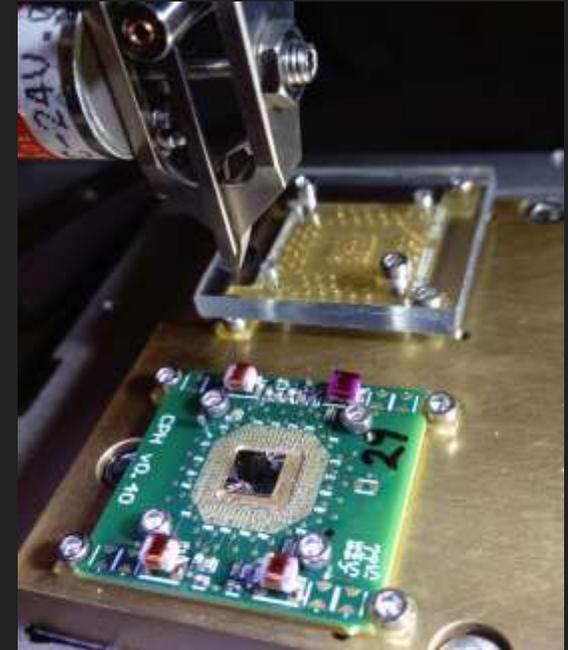
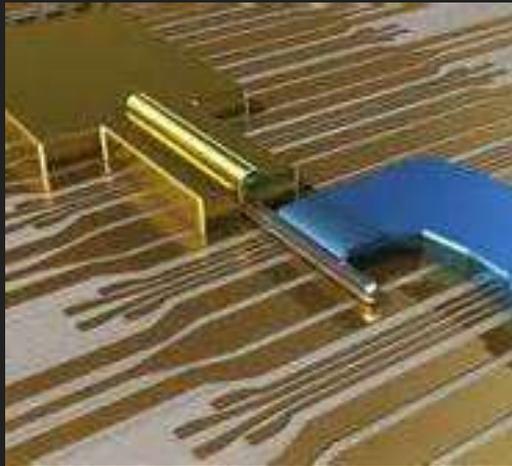
Carbon
capture



Materials
science

Machine
learning

Develop and deploy a scalable, commercial quantum system to solve today's unsolvable problems



2018

Microsoft's Unique Approach



Revolutionary topological approach



A global team



Scalable, end-to-end technology



Redmond



Santa Barbara



Delft



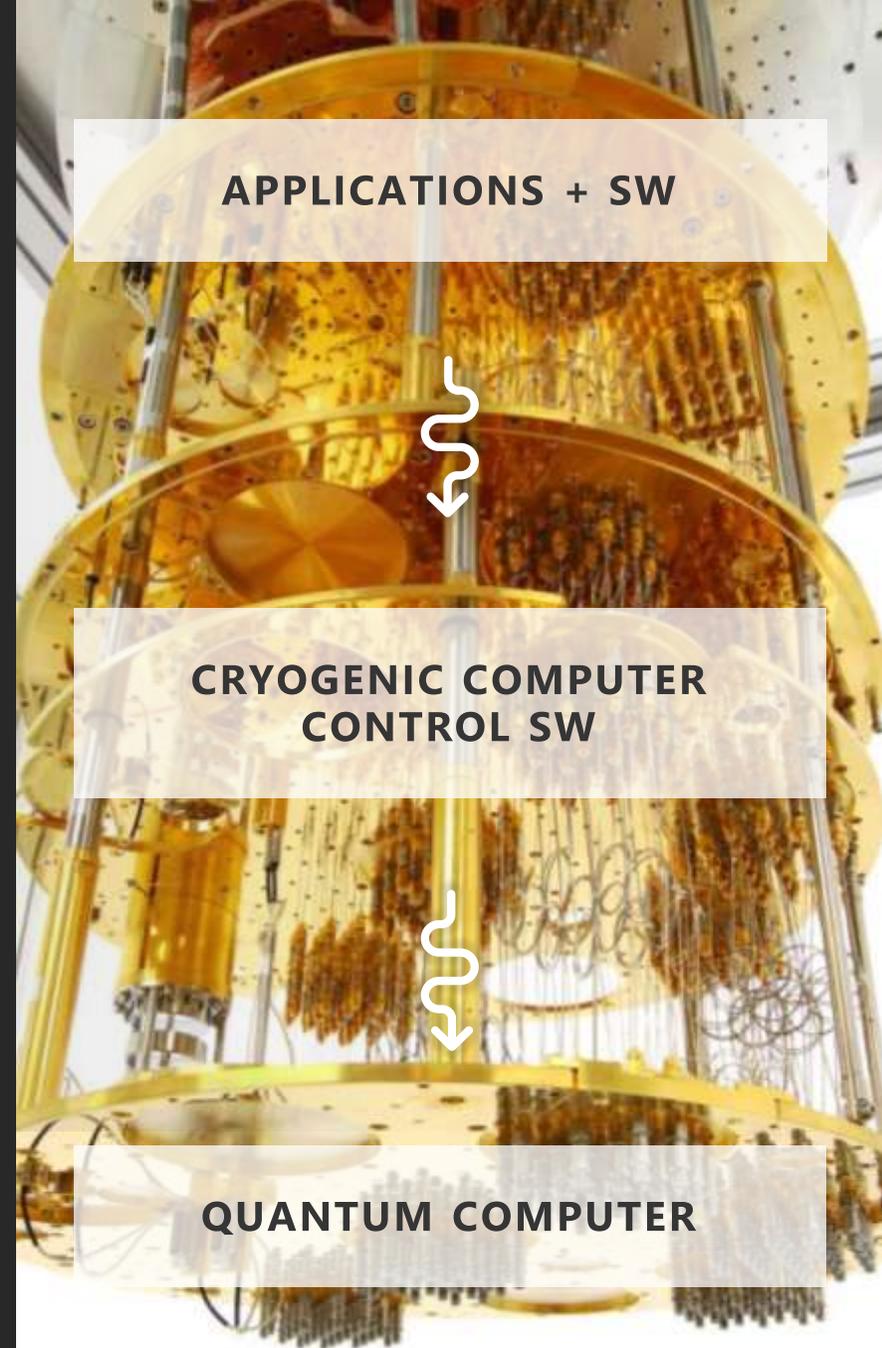
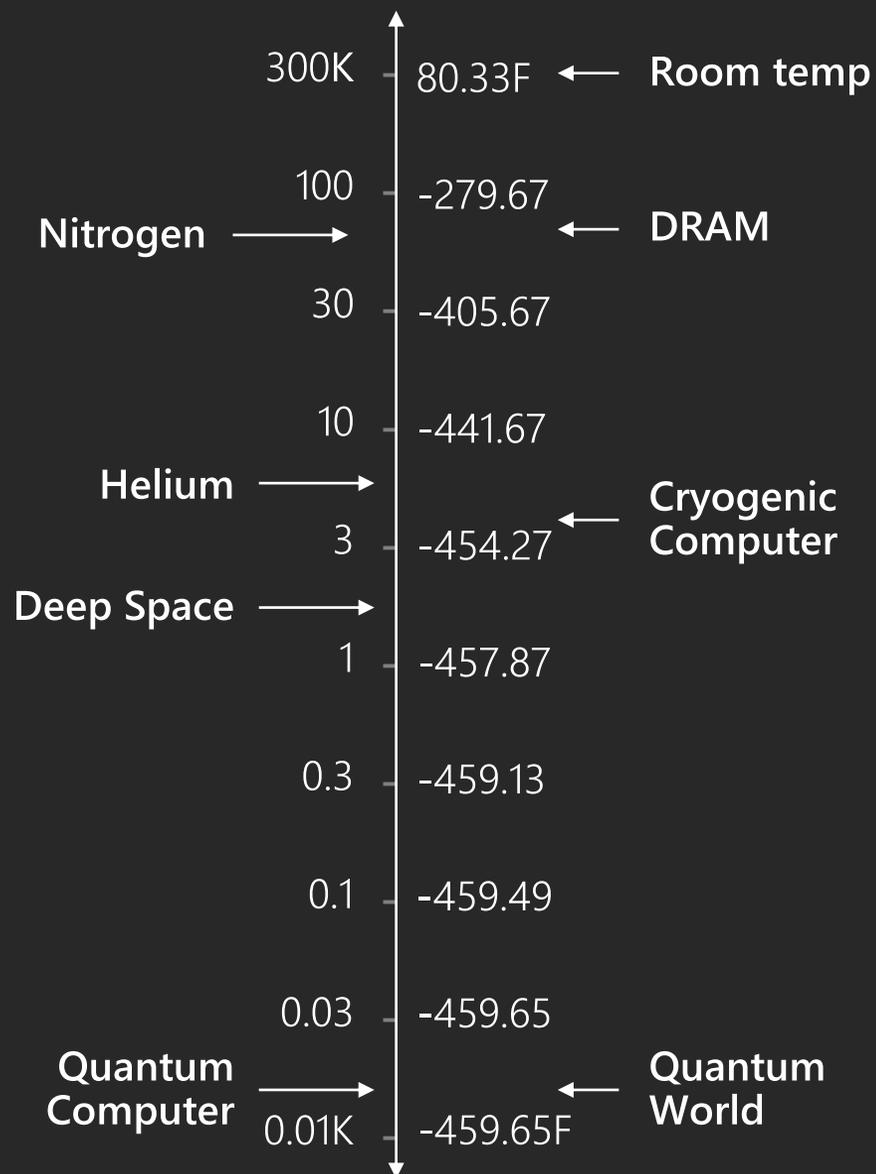
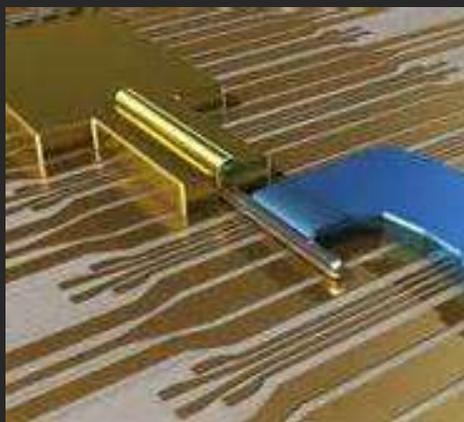
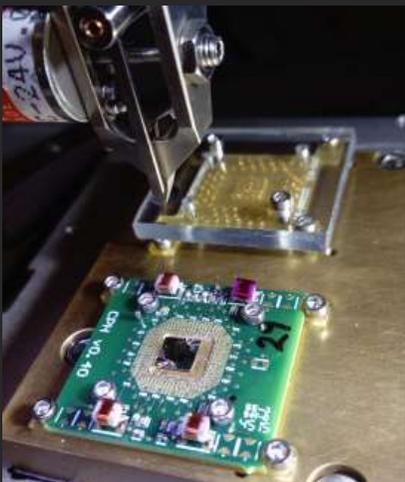
Copenhagen



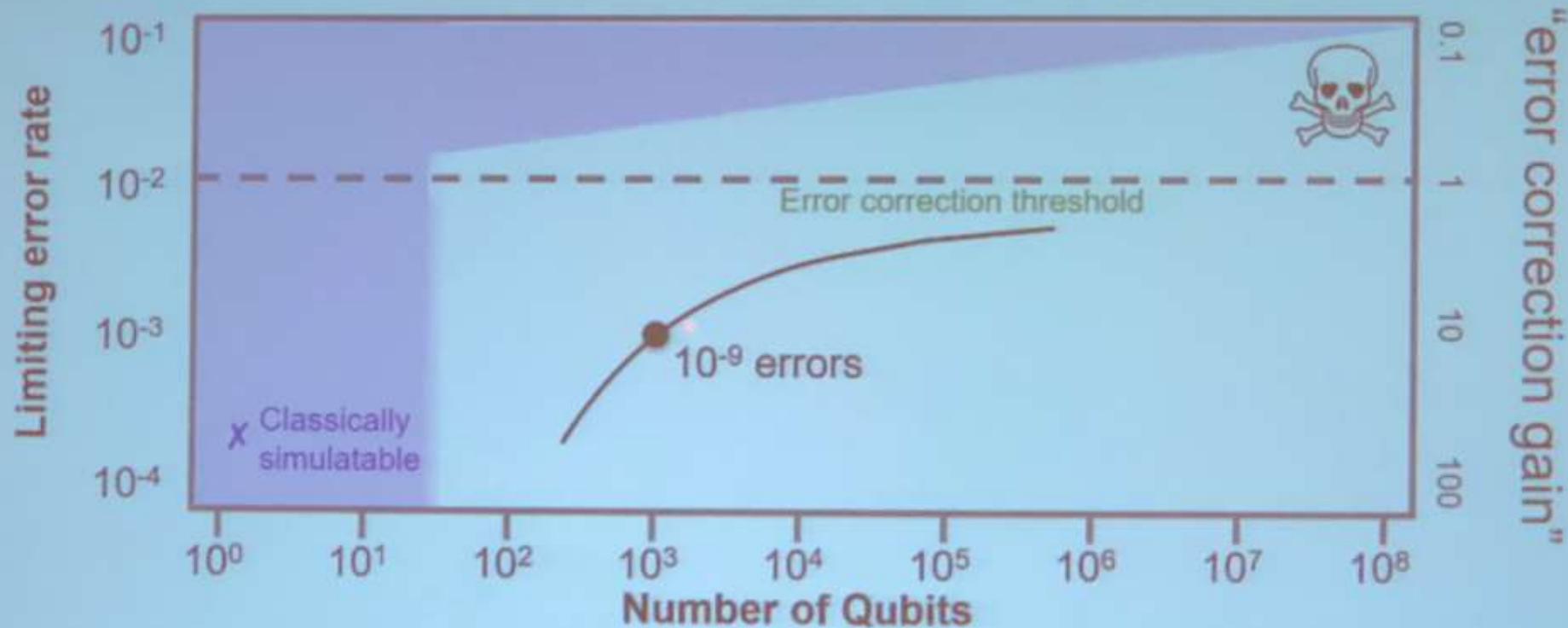
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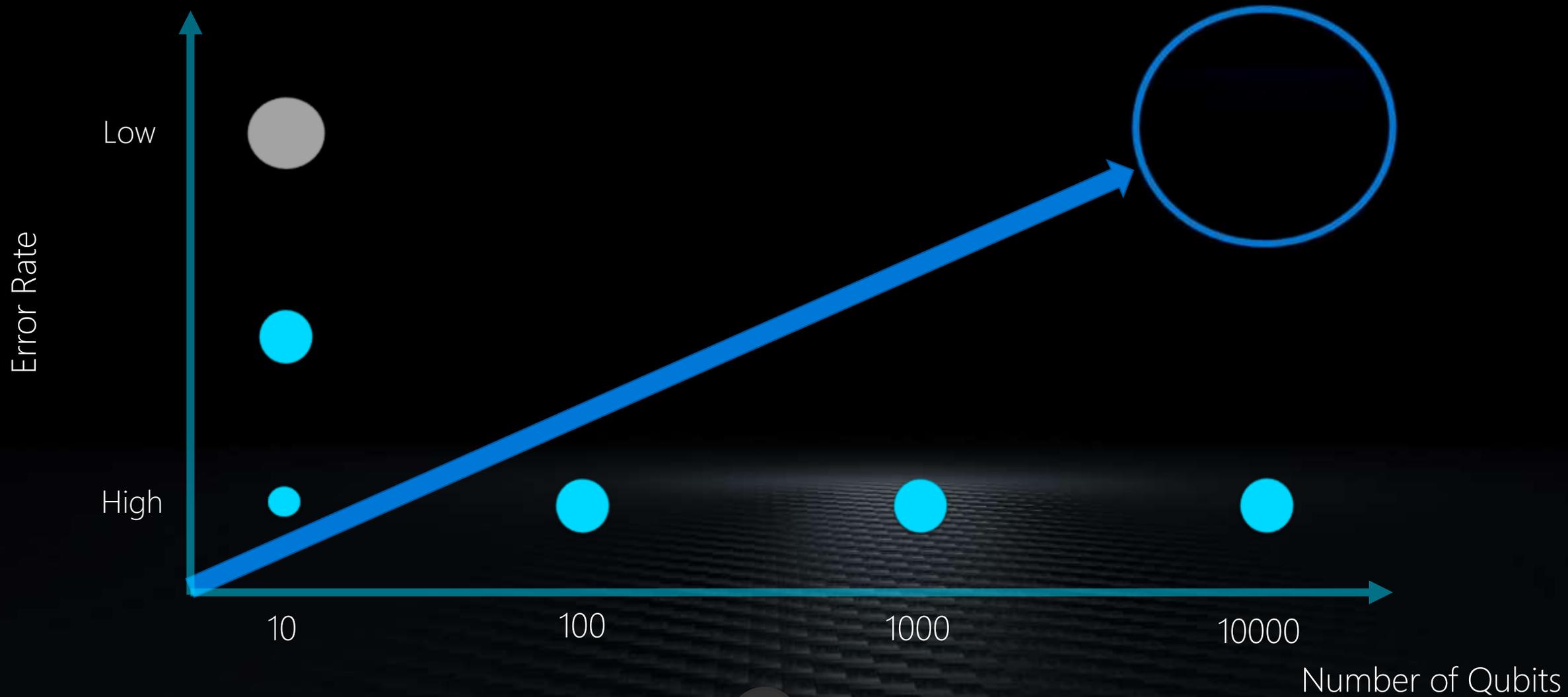


Lafayette



Need Both Quality and Quantity





All qubits are not created equal

Majorana Fermions

Predicted by Ettore Majorana
in 1937





Science

25 May 2012 | \$14

AAAS

“

MAJORANA PARTICLE
GLIMPSED IN LAB.

”

BBC NEWS

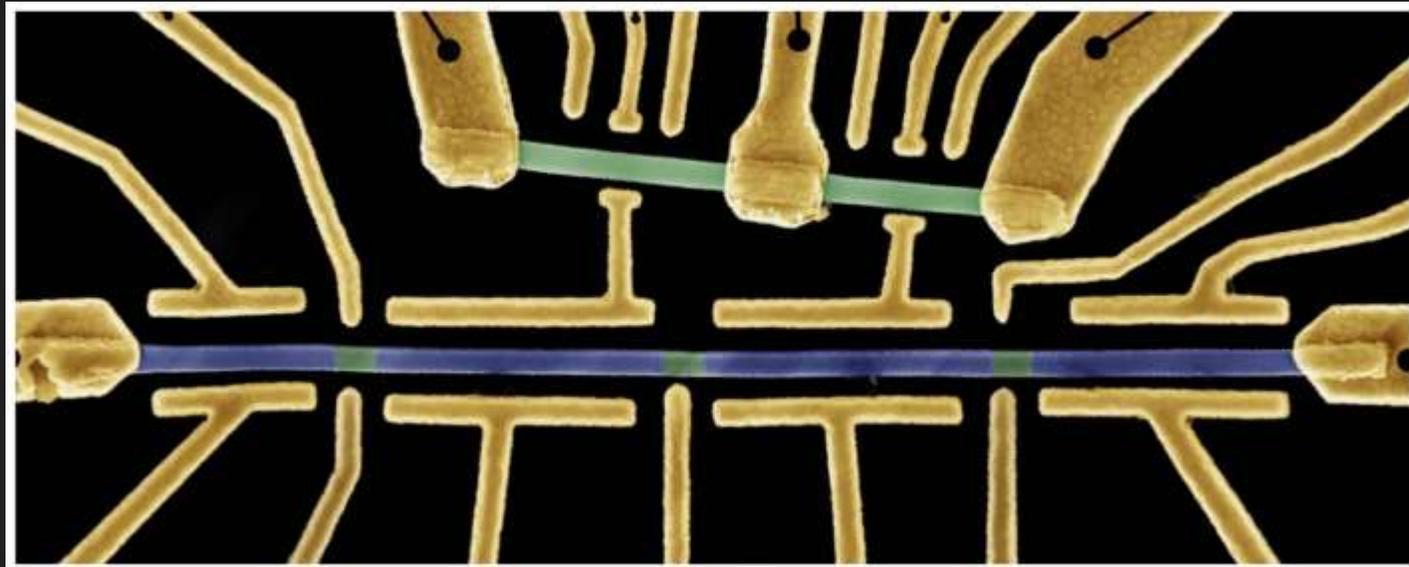


2012

Majorana qubits: split the information

Store a qubit in a superposition of 0 or 1 electrons

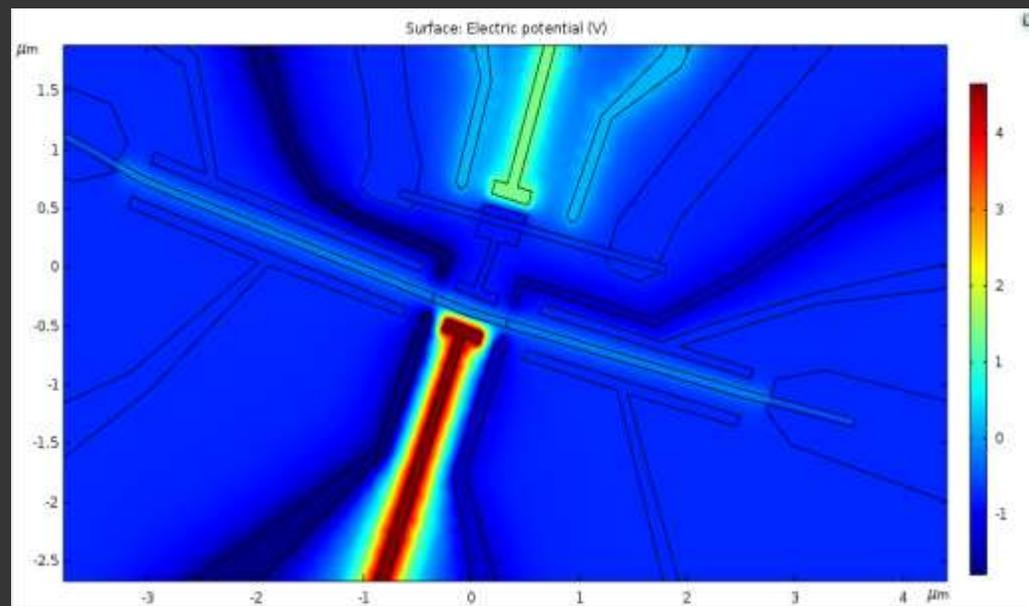
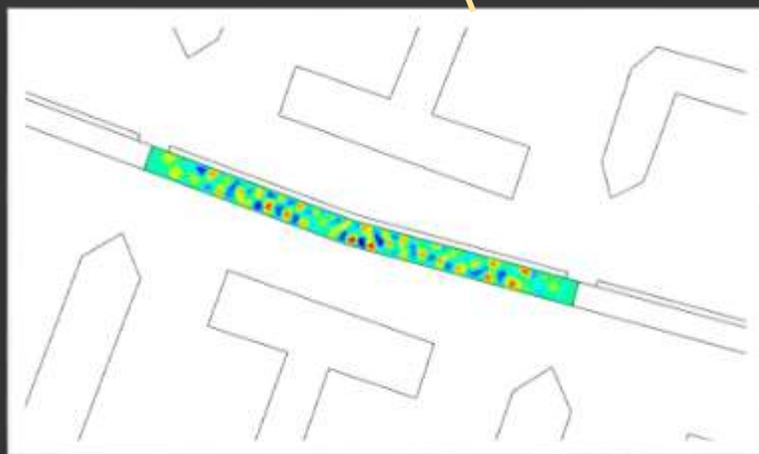
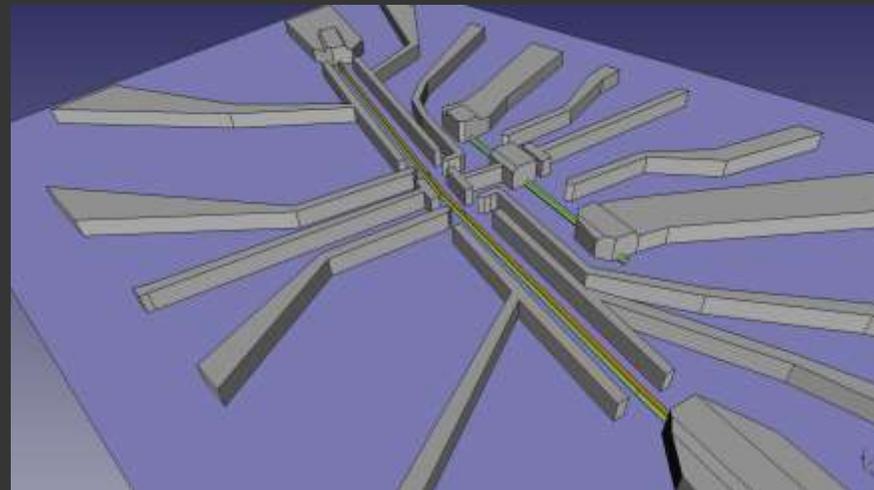
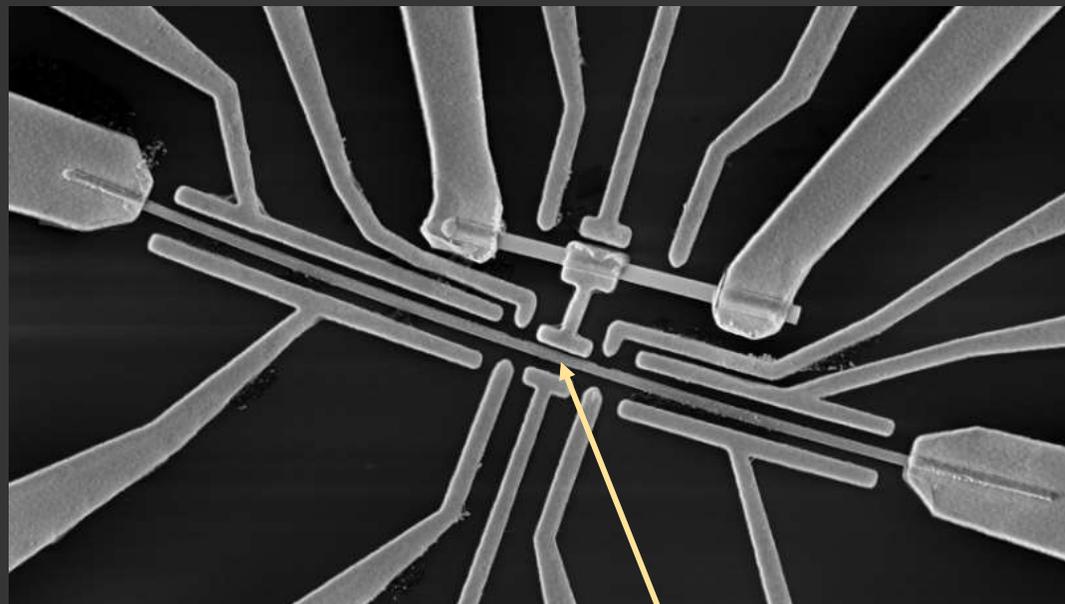
Split 0 or 1 electrons into two "Majorana" particles

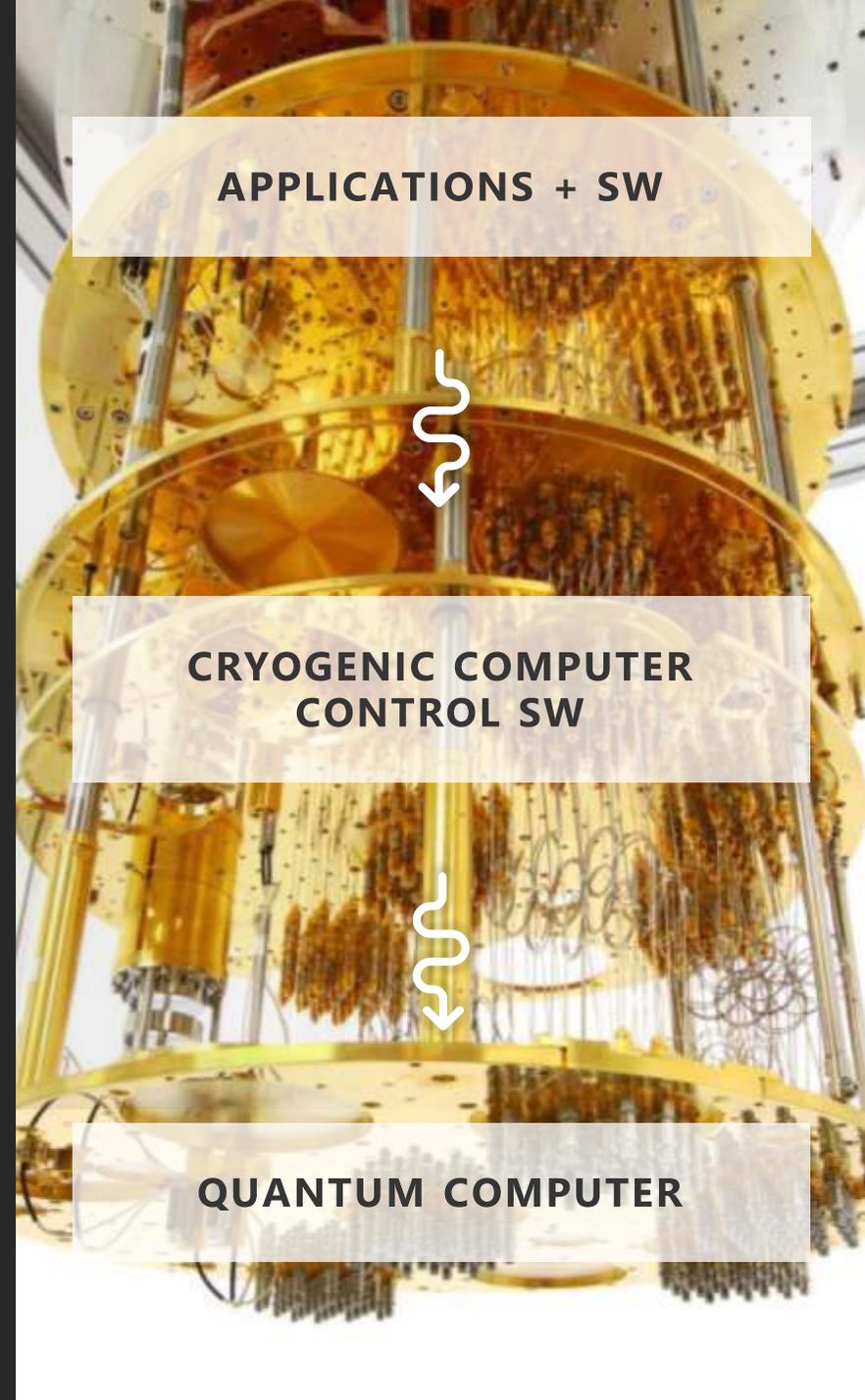
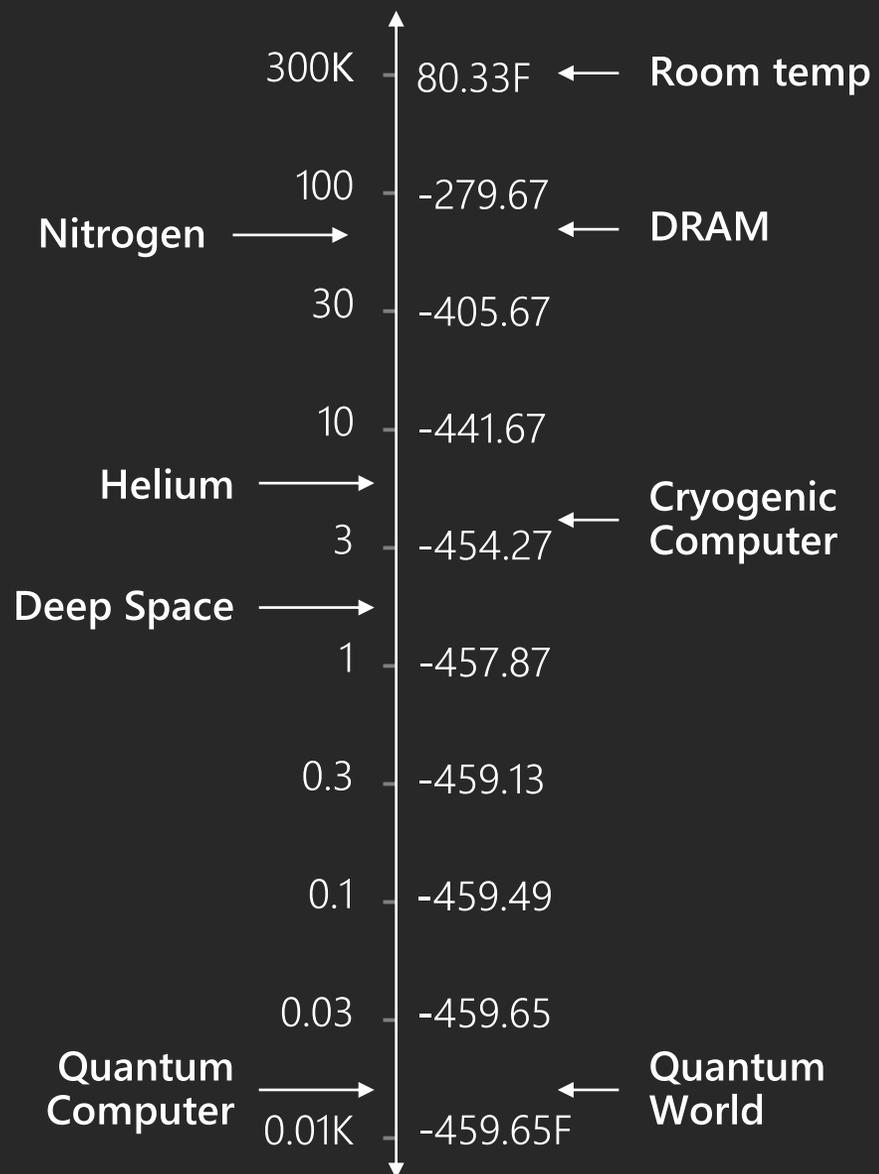
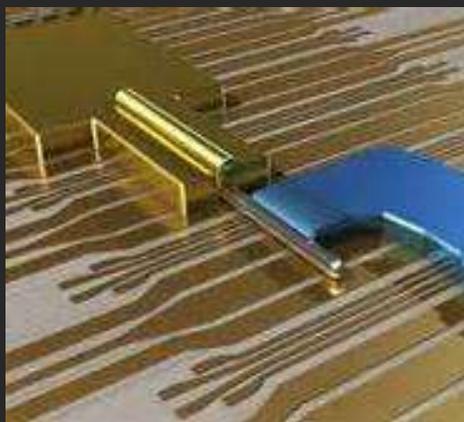
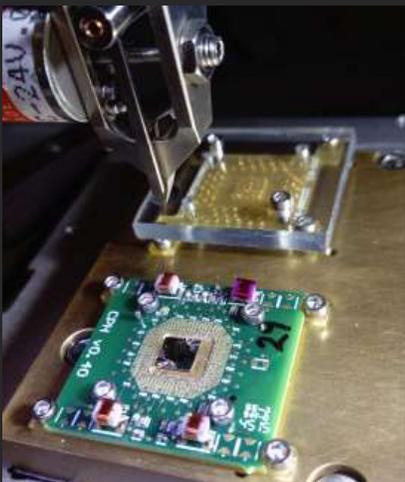




Inca Quipu

HPC support for qubit design





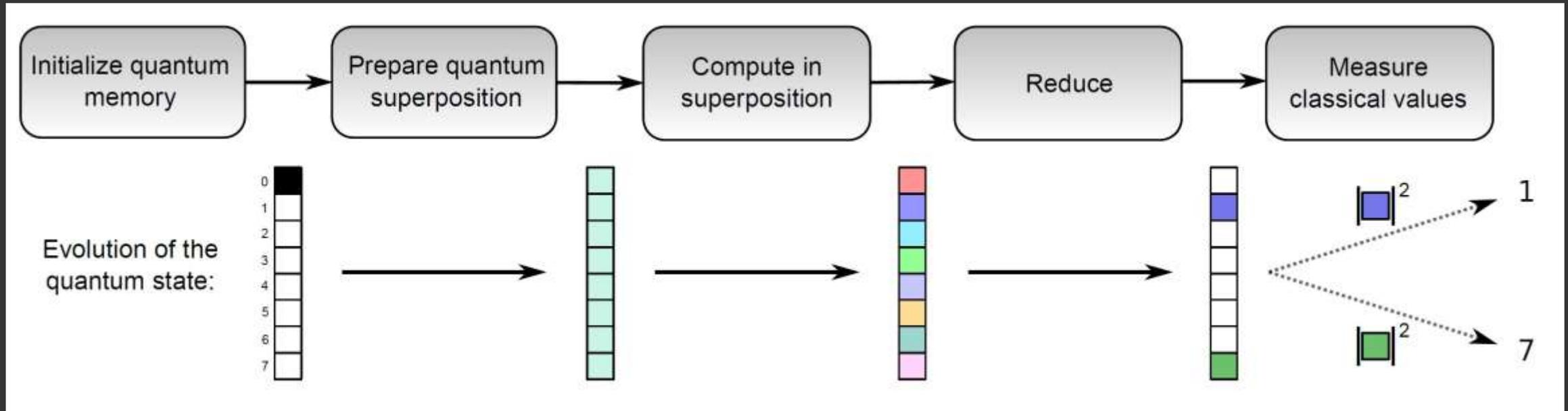
APPLICATIONS + SW

**CRYOGENIC COMPUTER
CONTROL SW**

QUANTUM COMPUTER

Computing in superposition

~~Quantum computers are fast because they operate on all values simultaneously~~



It's not that easy!

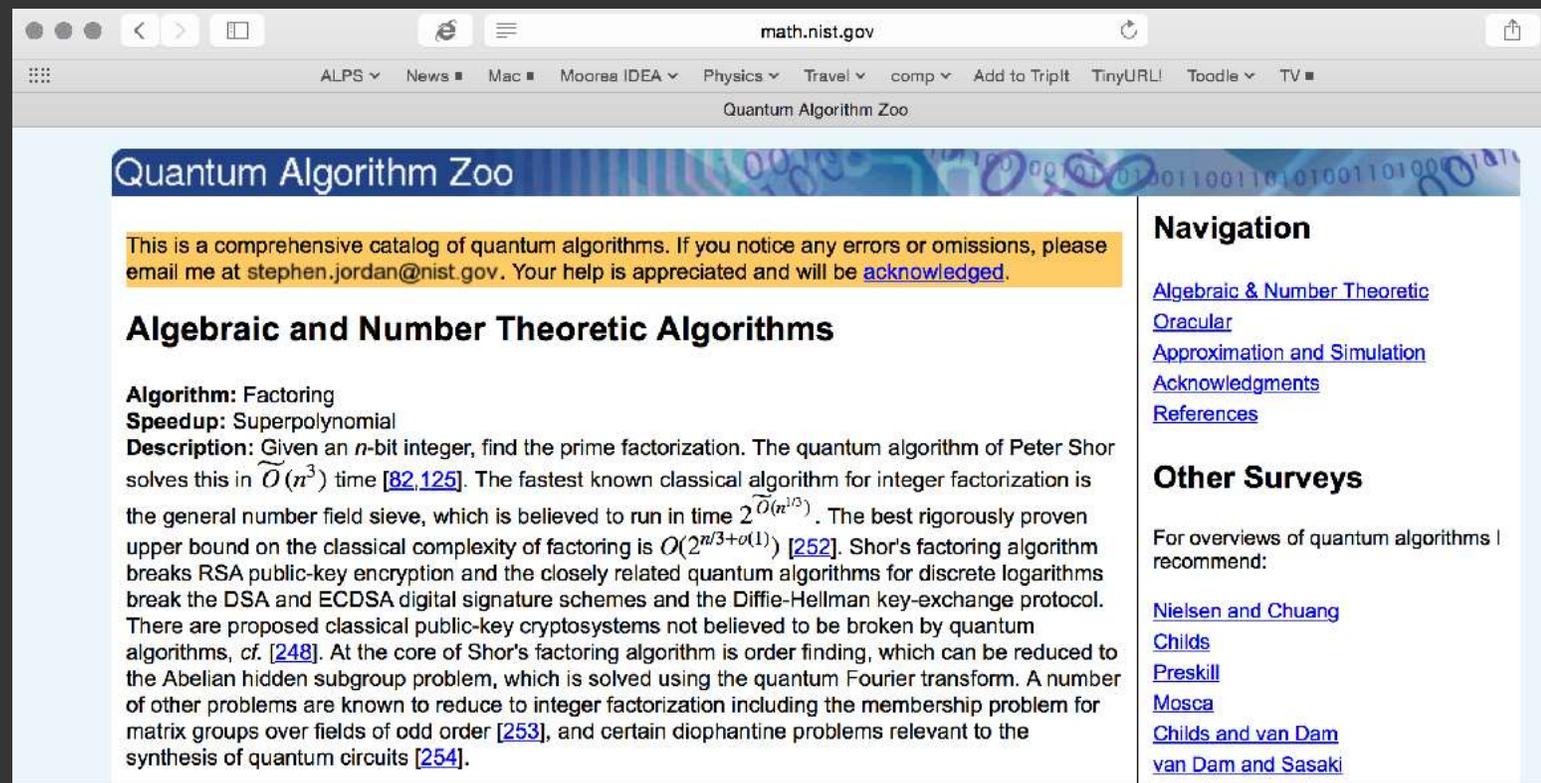
- Reading out the superposition only gives us a single random entry
- The crucial aspect of quantum algorithm design is finding good reduction operations that lead to (almost) deterministic answers

Developing quantum applications

1. Find quantum algorithm with quantum speedup

Quantum algorithms with quantum speedup

50+ quantum algorithms with quantum speedup, that is better asymptotic scaling than any classical computer



The screenshot shows a web browser window with the URL math.nist.gov. The page title is "Quantum Algorithm Zoo". A navigation menu includes links for ALPS, News, Mac, Moore's IDEA, Physics, Travel, comp, Add to TripIt, TinyURL!, Tooodle, and TV. The main content area features a blue header with the text "Quantum Algorithm Zoo" and a yellow box containing a notice: "This is a comprehensive catalog of quantum algorithms. If you notice any errors or omissions, please email me at stephen.jordan@nist.gov. Your help is appreciated and will be [acknowledged](#)." Below this is the section "Algebraic and Number Theoretic Algorithms". The entry for "Factoring" is shown, with details on its algorithm, speedup, and description. A right-hand sidebar contains "Navigation" links (Algebraic & Number Theoretic, Oracular, Approximation and Simulation, Acknowledgments, References) and "Other Surveys" (Nielsen and Chuang, Childs, Preskill, Mosca, Childs and van Dam, van Dam and Sasaki).

Quantum Algorithm Zoo

This is a comprehensive catalog of quantum algorithms. If you notice any errors or omissions, please email me at stephen.jordan@nist.gov. Your help is appreciated and will be [acknowledged](#).

Algebraic and Number Theoretic Algorithms

Algorithm: Factoring
Speedup: Superpolynomial
Description: Given an n -bit integer, find the prime factorization. The quantum algorithm of Peter Shor solves this in $\tilde{O}(n^3)$ time [82,125]. The fastest known classical algorithm for integer factorization is the general number field sieve, which is believed to run in time $2^{\tilde{O}(n^{1/3})}$. The best rigorously proven upper bound on the classical complexity of factoring is $O(2^{n^{3+o(1)}})$ [252]. Shor's factoring algorithm breaks RSA public-key encryption and the closely related quantum algorithms for discrete logarithms break the DSA and ECDSA digital signature schemes and the Diffie-Hellman key-exchange protocol. There are proposed classical public-key cryptosystems not believed to be broken by quantum algorithms, cf. [248]. At the core of Shor's factoring algorithm is order finding, which can be reduced to the Abelian hidden subgroup problem, which is solved using the quantum Fourier transform. A number of other problems are known to reduce to integer factorization including the membership problem for matrix groups over fields of odd order [253], and certain diophantine problems relevant to the synthesis of quantum circuits [254].

Navigation

- [Algebraic & Number Theoretic](#)
- [Oracular](#)
- [Approximation and Simulation](#)
- [Acknowledgments](#)
- [References](#)

Other Surveys

For overviews of quantum algorithms I recommend:

- [Nielsen and Chuang](#)
- [Childs](#)
- [Preskill](#)
- [Mosca](#)
- [Childs and van Dam](#)
- [van Dam and Sasaki](#)

<http://math.nist.gov/quantum/zoo/>

Developing quantum applications

quantum software engineers

1. Find quantum algorithm with quantum speedup

2. Confirm quantum speedup after implementing all subroutines and I/O

3. Optimize code until runtime is short enough

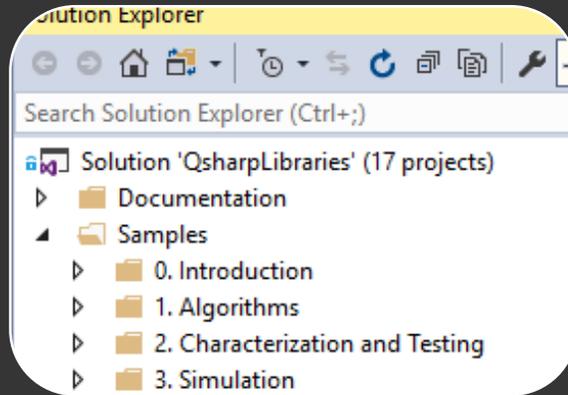
4. Embed into specific architecture and estimate resources

Introducing the Microsoft Quantum Development Kit

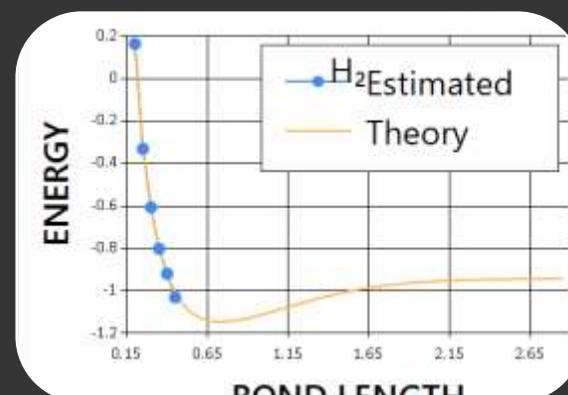
```
/// ## there
/// A qubit initially in the |0> state that v
/// the state of msg to.
operation Teleport(msg : Qubit, there : Qubit)
body {
    using (register = Qubit[1]) {
        // Ask for an auxillary qubit that
        // for teleportation.
        let here = register[0];

        // Create some entanglement that v
        H(here);
        CNOT(here, there);
    }
}
```

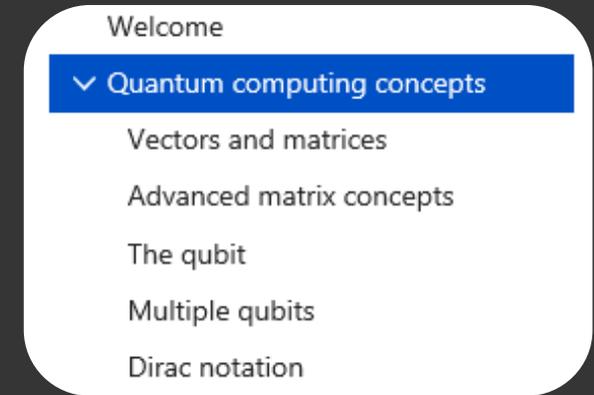
Quantum programming language Q#



Visual Studio integration and debugging



Local and cloud quantum simulation



Extensive open-source libraries, samples, documentation

Target machines

- **State-of-the-Art Local Simulator**
 - Simulate 30 qubits in 16 GB
 - Run locally on your PC
- **State-of-the-Art Azure Simulator**
 - Simulate more than 40 qubits
 - Run in Azure
- **Trace Simulator**
 - Determine resource costs of quantum program
 - Scale to large algorithms and numbers of qubits
- **Quantum Computer**



Distributed simulator using MPI

2017: supercomputer



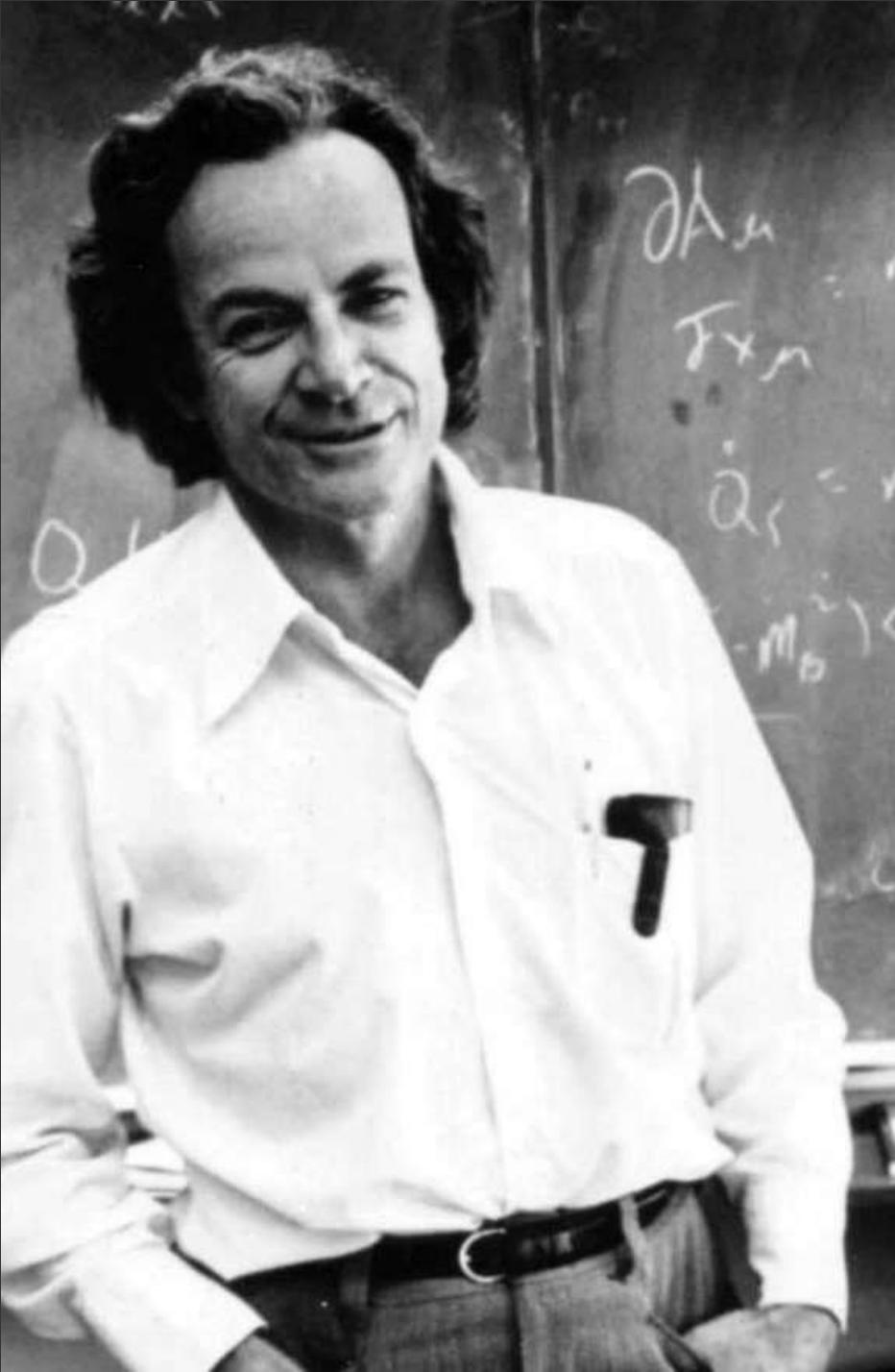
2018: cloud



MPI-based distributed simulator, similar to that of ETH students D. Steiger and T. Häner

More than 30x reduction in network traffic compared to other simulators (SC'17 paper)

No need to buy a supercomputer: available as Azure HPC cloud services up to ≈ 43 qubits



International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

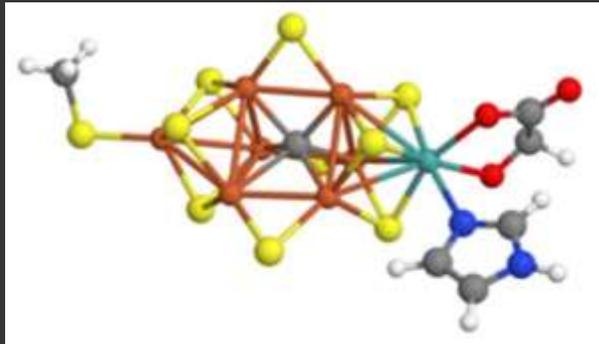
Received May 7, 1981

Feynman proposed to use quantum computers to simulate quantum physics

We can surpass the best classical computers with only 50 qubits!

There are many classical algorithms for simulating quantum systems, but they all have limited applicability.

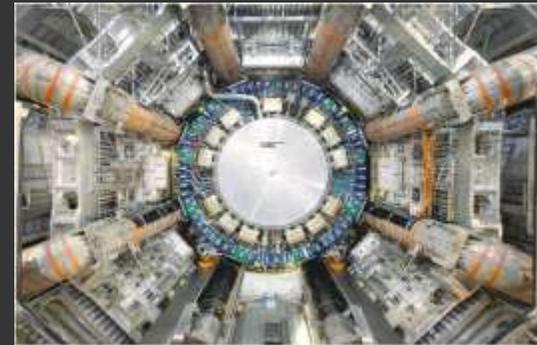
Chemistry



Materials

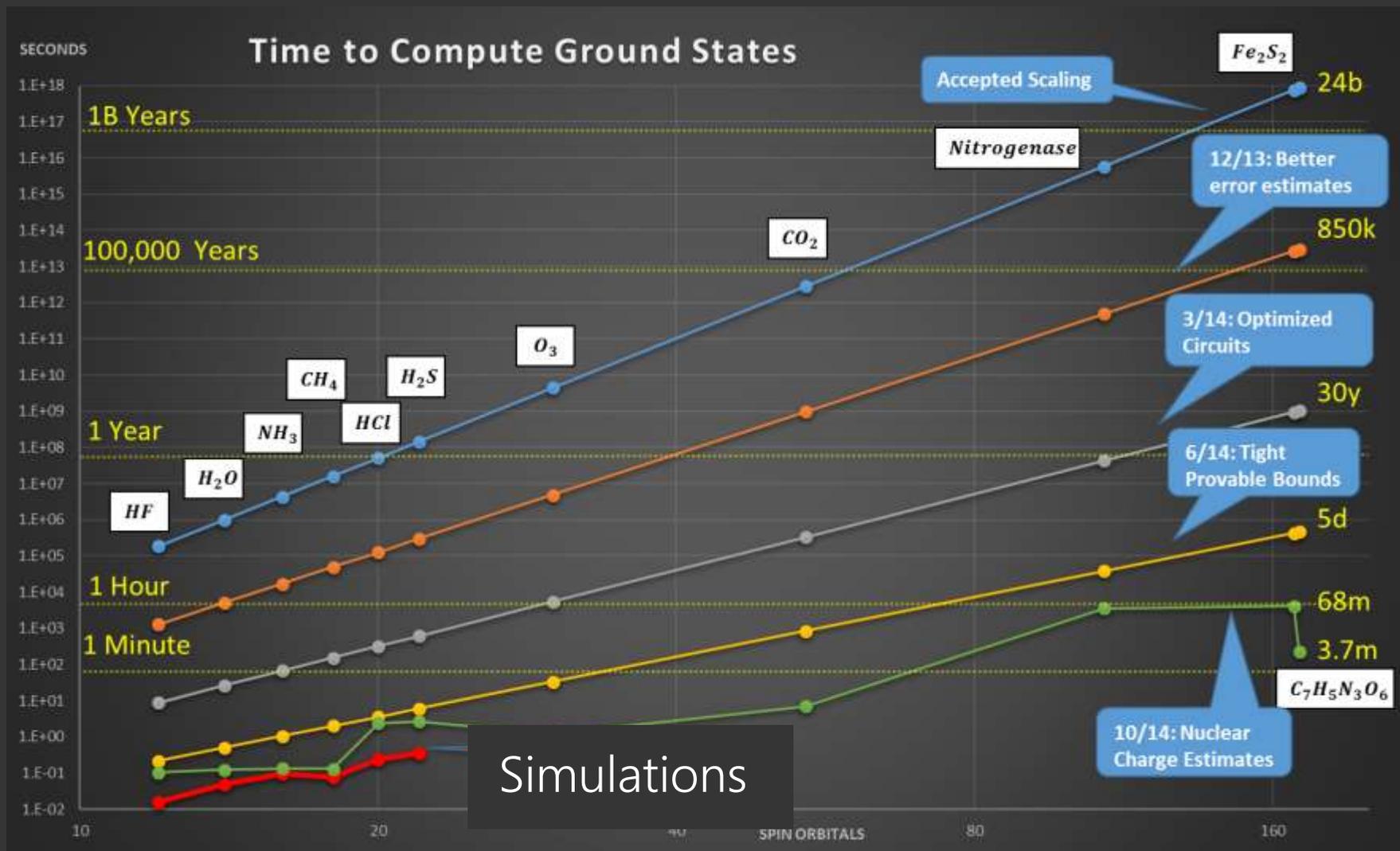


Nuclear and Particle Physics



Many problems are out of reach even for exascale supercomputers but doable on quantum computers.

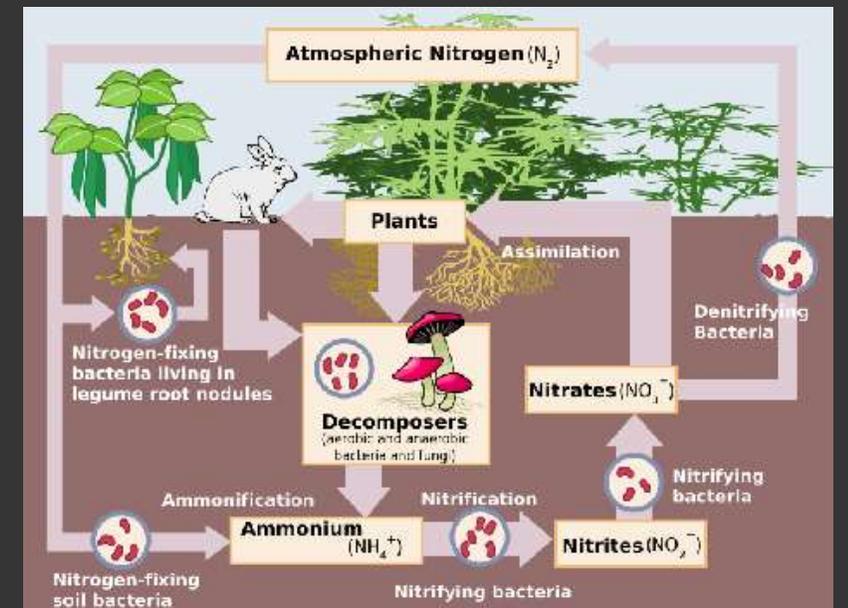
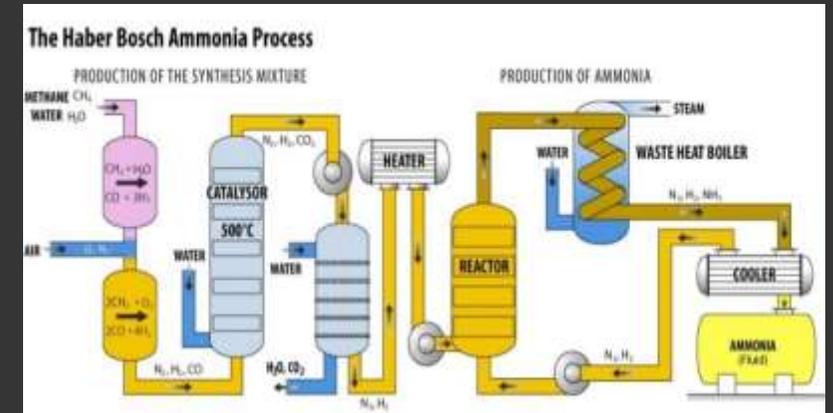
Simulation Evidence



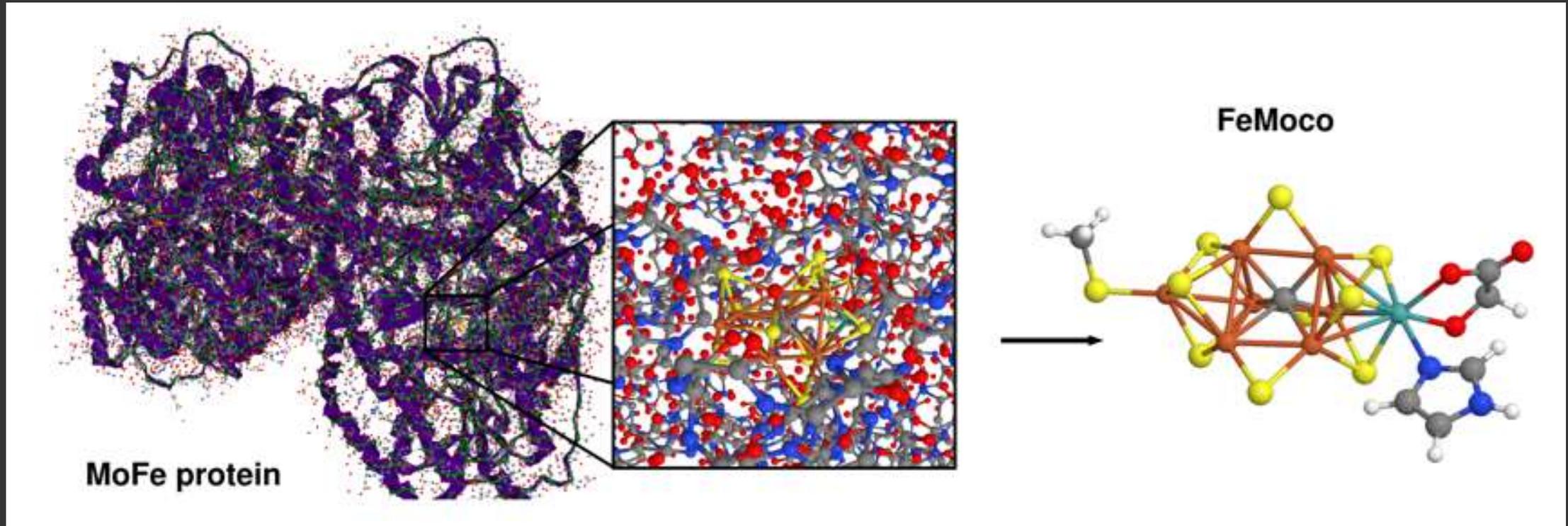
Nitrogen fixation

Fertilizer production using Haber-Bosch process needs 3% of the world's natural gas production

But bacteria can do it in the soil without a huge factory!



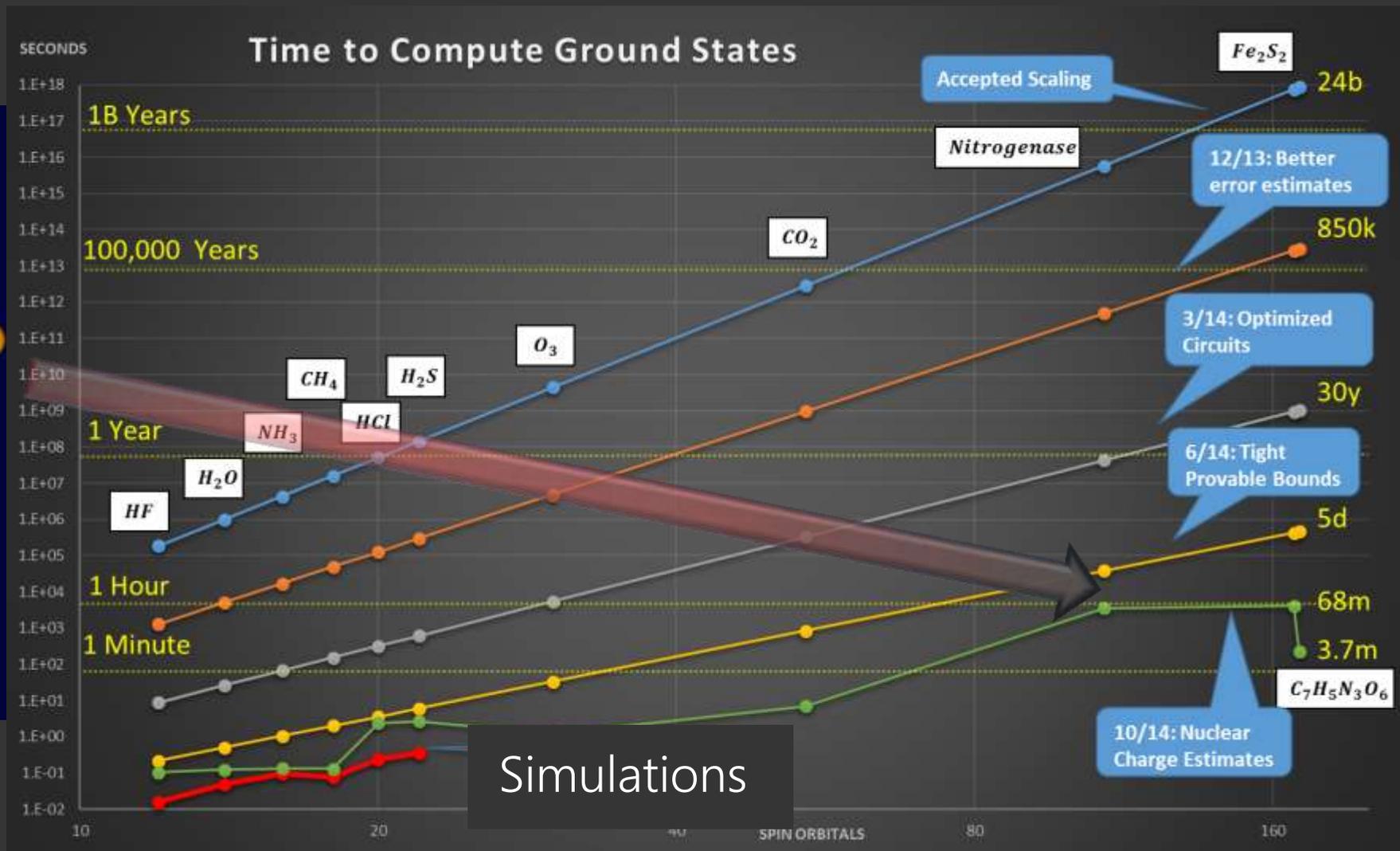
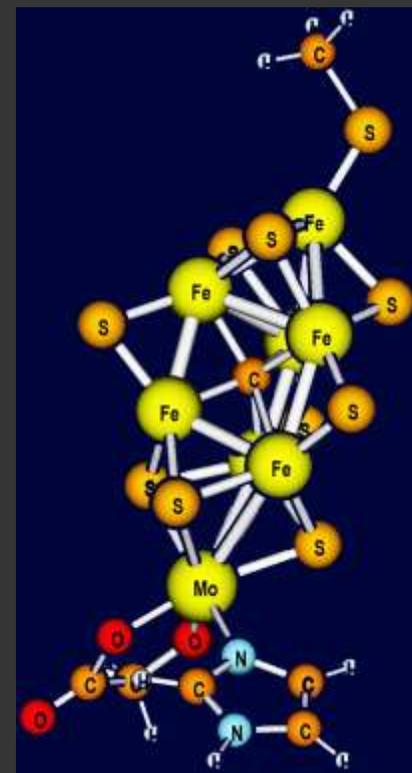
Understanding biological nitrogen fixation



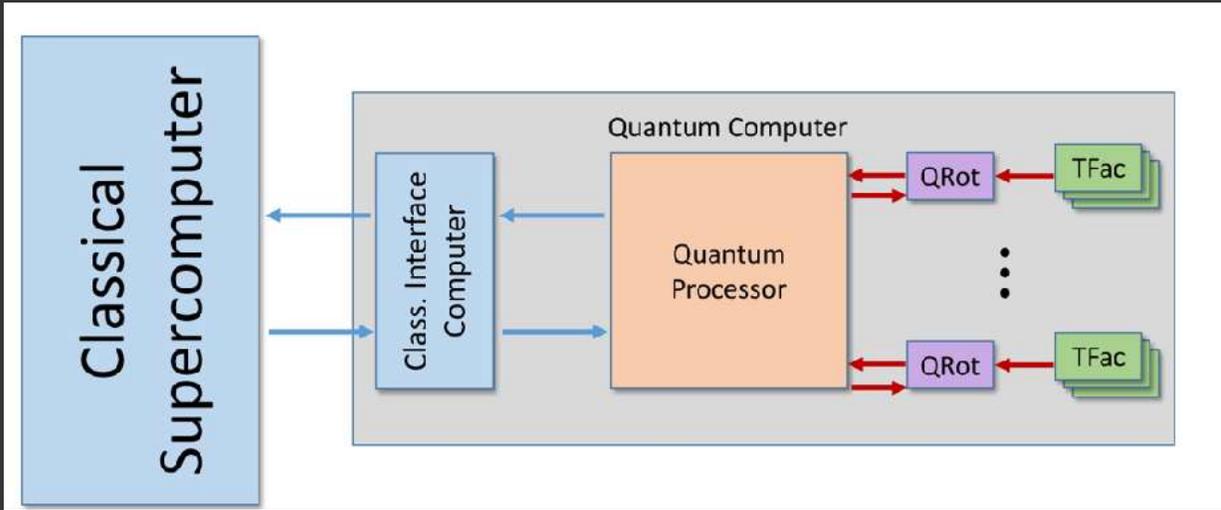
Intractable on classical supercomputers

But a 200-qubit quantum computer will let us understand it

Simulation Evidence



Total cost with quantum error correction

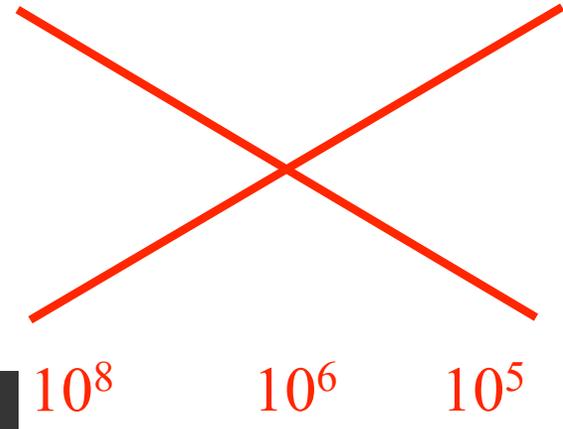


We need better qubits!

Better algorithms!

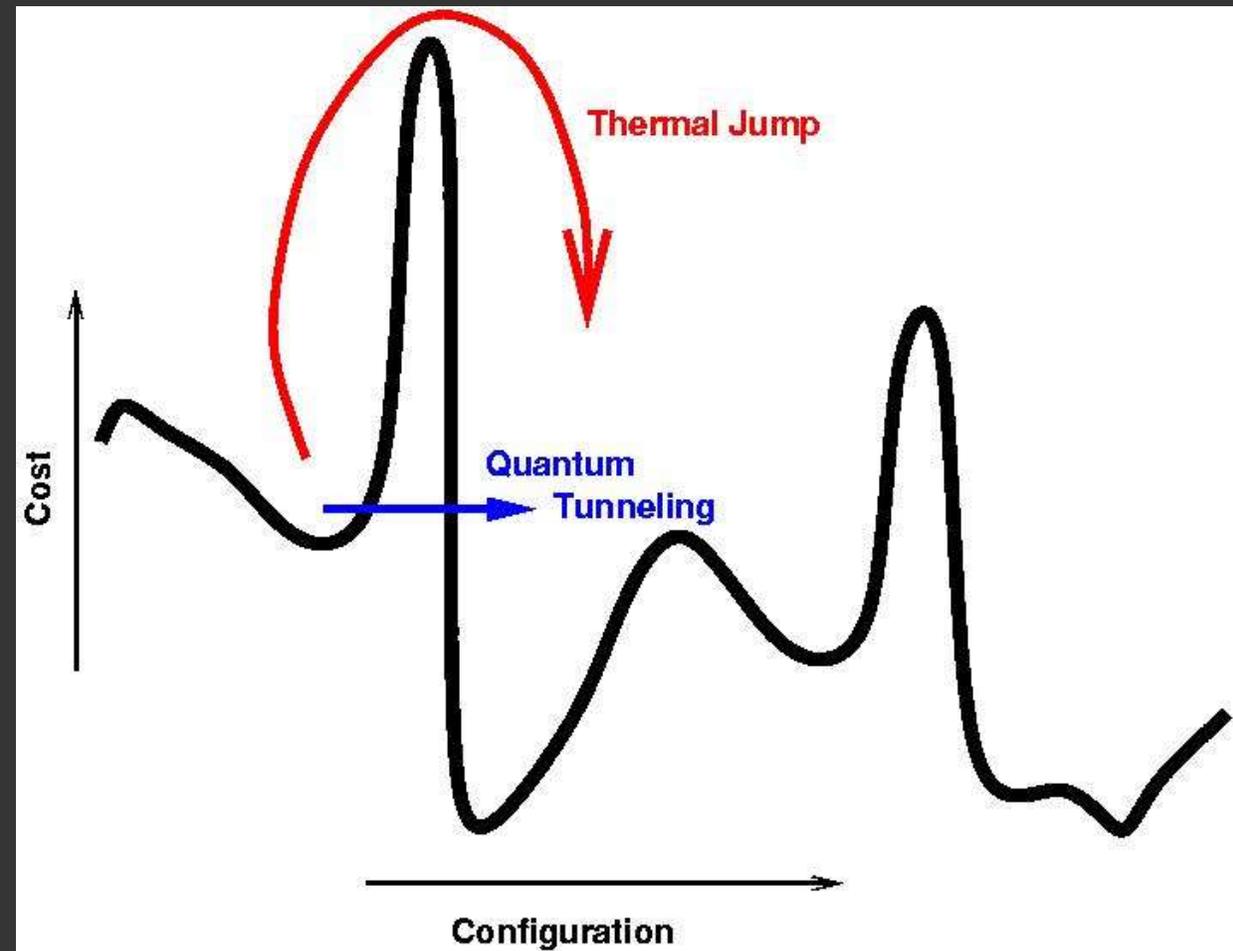
Better quantum error correction
(Hastings & Haah)

	Nested rotations		
Error Rate	10^{-3}	10^{-6}	10^{-9}
Required code distance	37,17	9	5
Quantum processor			
Logical qubits	109		



Quantum optimization

Use quantum effects to escape local minima by tunneling under a barrier



Early value: quantum inspired optimization

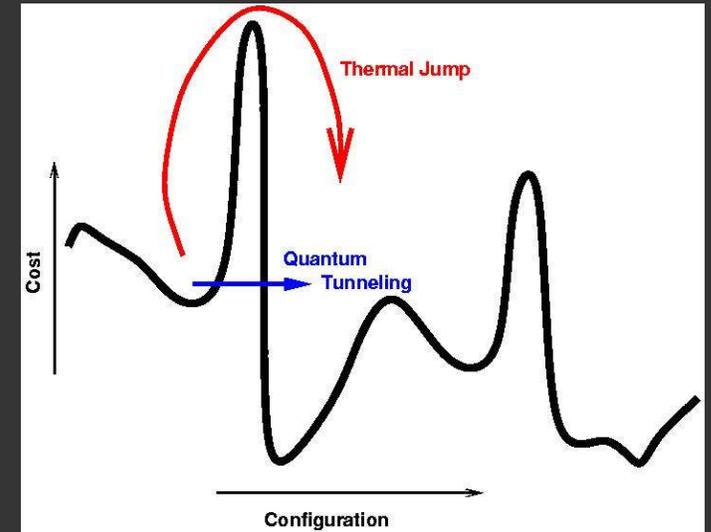
Mimic quantum tunneling
on classical computers today!

Research into [quantum algorithms](#) often
uncovers new [classical algorithms](#)

Better optimization methods

Better training algorithms

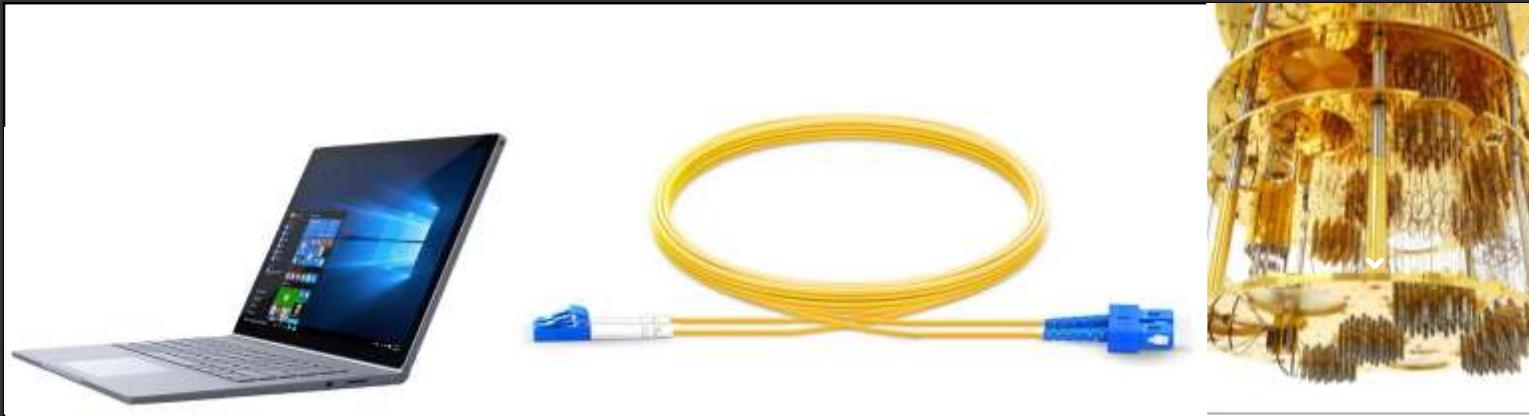
Better models for machine learning



Will cloud service respect privacy of my data?
Will the computation be implemented correctly?

Blind quantum computation can certify privacy and correctness of delegated computation.

(Broadbent, Fitzsimons, Kashefi, 2009)



It is time to start writing quantum software

Invent new quantum algorithms

Explore quantum applications

Determine resources and optimize quantum code

Co-design quantum hardware and software



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operation [] H (qubit q1, qubit q2) [  
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