Parallel Pseudorandom Number Generators for Large Parallel Computations

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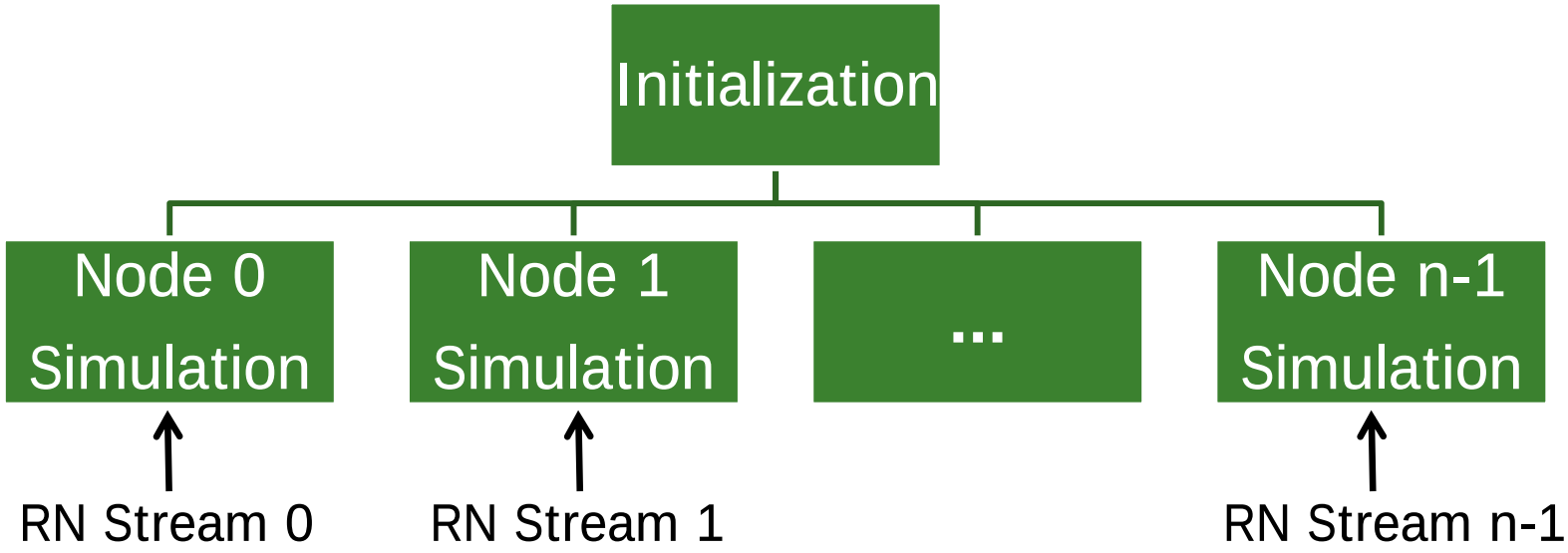
Terminology

- RN: pseudorandom number
- RN stream: a sequence of RNs
- Cycle: the maximum number of RNs a stream gives before repeating the starting sequence of RNs

- RNG: pseudorandom number generator
- PRNG: parallel pseudorandom number generator
  - Provides multiple distinct RN streams

- Correlation: how predictable a set of RNs is given another set of RNs
  - Interstream and intrastream correlations

Mobile Ad Hoc Network Simulator



Node Simulation Pseudocode

```
// node i, i varies 0 … n-1
While (1) {
  …
  Delta_X = RNstream(i)*MAX_DELTA;
  Delta_Y = RNstream(i)*MAX_DELTA;
  Move node to (Current_X+Delta_X, Current_Y+Delta_Y)
  …
  // Received a broadcast packet. Retransmit it selectively
  Process the packet
  if (RNstream(i) < ReTX_Probability)  {
    Jitter = RNstream(i)*MAX_JITTER;
    Broadcast Message at  Current_Time + Jitter
  }
  …
}
```

For better randomness, the RNs used for Jitter, node movement, re-broadcast decision, … should be from different RN streams

— tedious to manage at the application level

Results Obtained

- **Context-aware parallel random number generator (CPRNG)** **[Patent pending]**
  - Virtually unlimited number of random number streams, each stream with a large cycle
  - Provides distinct streams based on the location of calls for RNs (user selectable; no app. recoding)
    - Reduces the impact of intrastream correlations
  - Dynamically allocates additional RN streams beyond any upper limit specified
    - Some large, complex simulations require unpredictable and very large number of RN streams for subtasks

Results Obtained

- **Interstream Correlation (ISC) Test** **[Pat. pending]**
  - Evaluates correlations among parallel streams and gives a single number as quality metric
    - The first statistical test of its kind
    - The current statistical tests are 15-150 single-stream tests for intrastream correlations
      - No single quality metric; a vector of pass/fail data
      - Interstream correlations: interleave streams & apply s-s tests
  - Integrates with CPRNG to provide the quality metric continually as the application consumes RNs
    - Not feasible with the current test batteries

Outline

- Related work
- Context-aware parallel pseudorandom number generator
- Interstream correlation test
- Summary and further work

Sequential RNGs

- Linear congruential generator (LCG) $x_n = a x_{n-1} + b \pmod{2^{64}},\ n > 1$
  - Specify a, b, and the initial value x_0
  - Parallel streams are obtained by choosing different values for a
  - 48-bit versions in Unix systems: drand48()

- Recursion with carry (RWC): $c_n \,||\, x_n = a_{-1} x_{n-1} + \cdots + a_{-5} x_{n-5} + c_{n-1} \pmod{2^{32}}$
  - G. Marsaglia, Diehard package, FSU
  - RWC is used to seed the parallel RNGs we implemented

- Mersenne Twister (MT): $x_{k+n} = x_{k+m} + (x_k^{upper} \,||\, x_{k+1}^{lower})A,\ n > m$
  - A is a 64x64 bit matrix
  - x_{k+n} is further multiplied by a bit matrix T to improve randomness
  - Very popular; GPU version, MTGP, by Nvidia
  - Matsumoto & Nishimura [ACM TOMACS 1998]

Parameterized Parallel RNGs

- ALFG $x_n = x_{n-k} + x_{n-l} \pmod{2^{64}},\ 0 < k < l < n$
  - Additive lagged Fibonacci generator
  - Initialization requires specification of $x_0, \ldots, x_{l-1}$
  - $2^{63*(l-1)}$ distinct RN streams
  - Each with a cycle of length $2^{63}*(2^l-1)$
  - Lag l=1279 or larger is used to minimize intra- and inter-stream correlations
  - A canonical form to initialize ALFG streams is known

- MLFG $x_n = x_{n-k} * x_{n-l} \pmod{2^{64}},\ 0 < k < l < n$
  - Multiplicative lagged Fibonacci generator
  - $2^{61*16} = 2^{976}$ distinct RN streams with lag l=17
  - Each with a cycle of length $2^{61}*(2^{17}-1) \approx 2^{78}$
  - Lag l=17 is sufficient to get high quality RN streams

Newer RNGs

- Cryptography-based RNGs
  - Use the scrambling techniques from cryptography to generate random numbers from simple sequences
- GPU friendly RNGs
  - Nvidia adapted MT for GPU implementation and offered as part of CUDA library
  - Parameterized RNGs

SPRNG: Parallel RNG and Test Package

- A. Srinivasan et al. ACM TOMS 2000, Parallel Computing 2003 and 2004
- Provides
  - 6 PRNGs: ALFG, MLFG, two LCGs and two other generators
  - Several single stream tests
    - Interleaves multiple parallel RN streams to form a sequential stream and applies sequential tests for interstream correlation checks
  - Ising model (in Physics) simulation codes using Metropolis and Wolff algorithms
    - Exact solutions are known so that the simulation error can be estimated
- Used for initial implementation of CPRNG

Test Packages

- Statistical tests to analyze intrastream correlations of a single stream
  - Dieharder (Brown, Duke University)
  - TU01 (‘Ecuyer & Blouin, ACM TOMS, 2007)

- Application-based testing
  - 2-D Ising model simulations
    - Exact theoretical results known
    - Simulations are used to evaluate the quality of RNGs
    - Two algorithms for simulations: Metropolis and Wolff

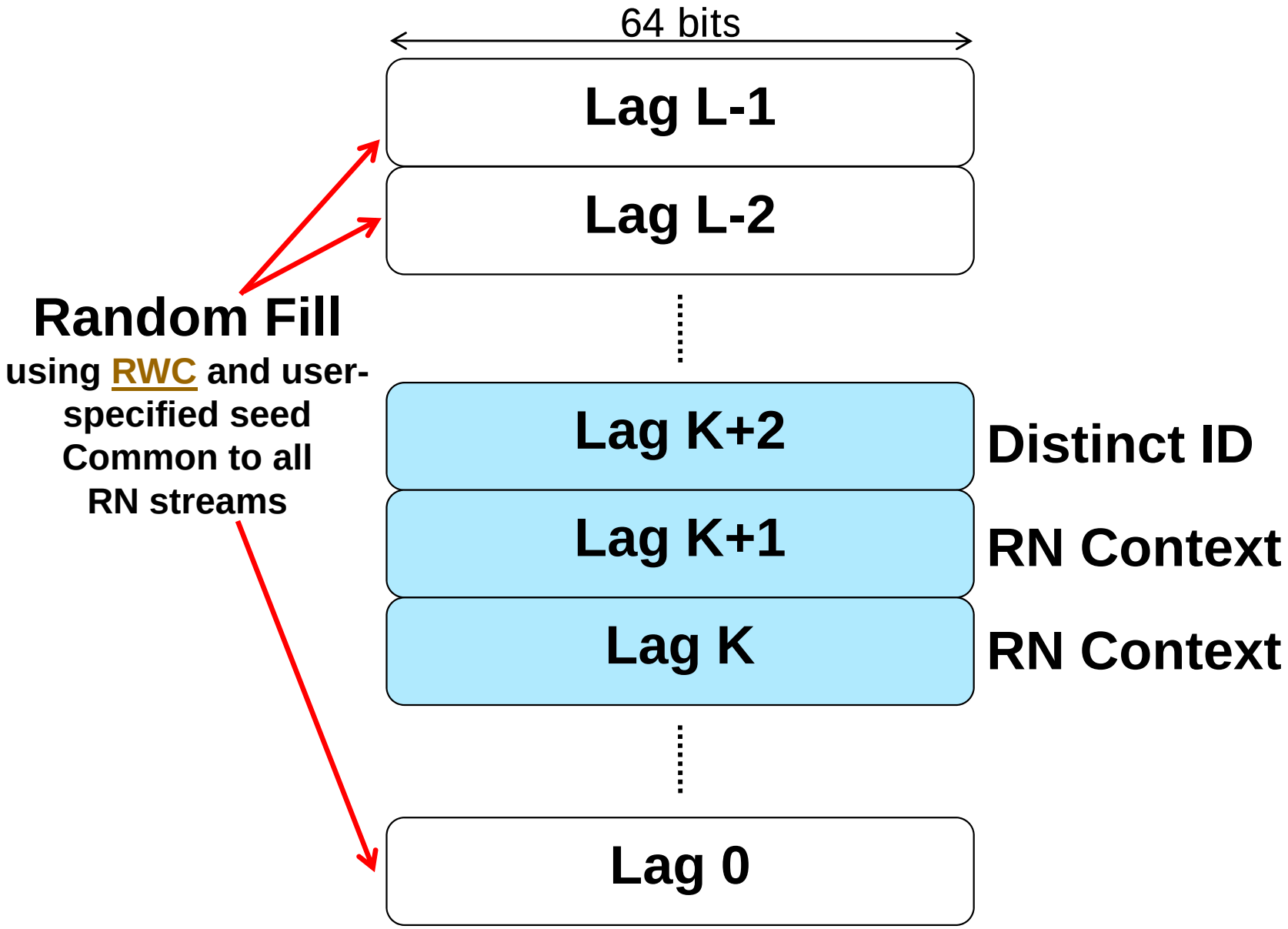
Context-aware parallel pseudorandom number generator (CPRNG)

Based on the Fibonacci recurrence:

$$x_n = x_{n-k} * x_{n-l} \pmod{2^{64}},\ 0 < k < l < n$$

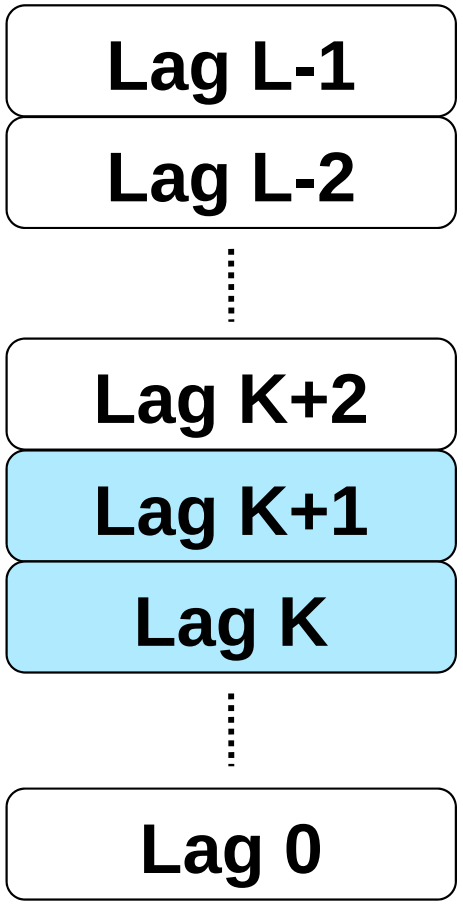
Implementation: an include file + a small library module

CPRNG Initialization of RN Streams

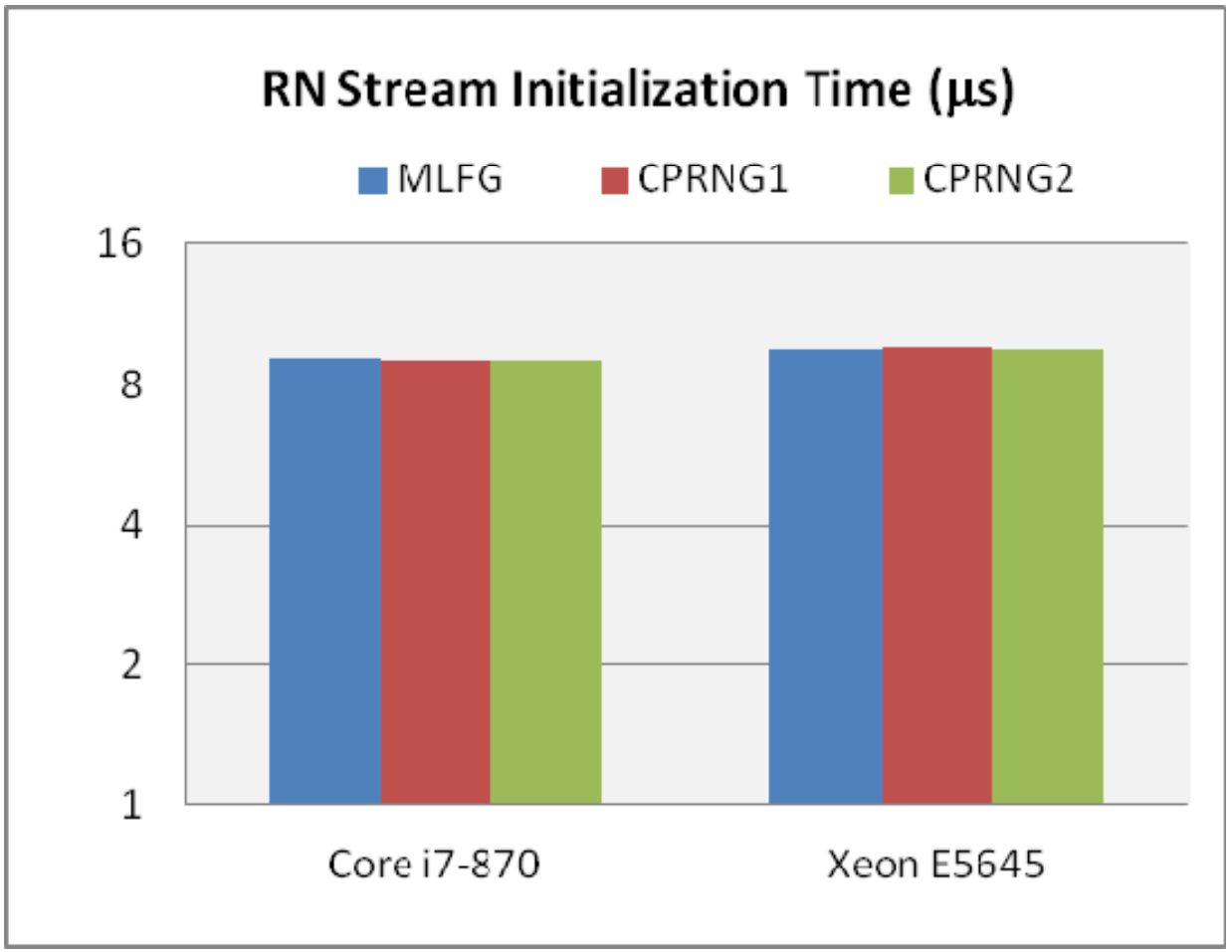


CPRNG Initialization: RN Context

- Calls for RNs with the same stream ID, but from different locations, result in the use of distinct RN streams
- Random number context consists of
  - Stream identifier: 0, ..., total_str-1
  - Program location: return address of the function call to RNG, or program line number and file name
  - Optional: process ID, thread ID, iteration number



RN Stream Initialization Time

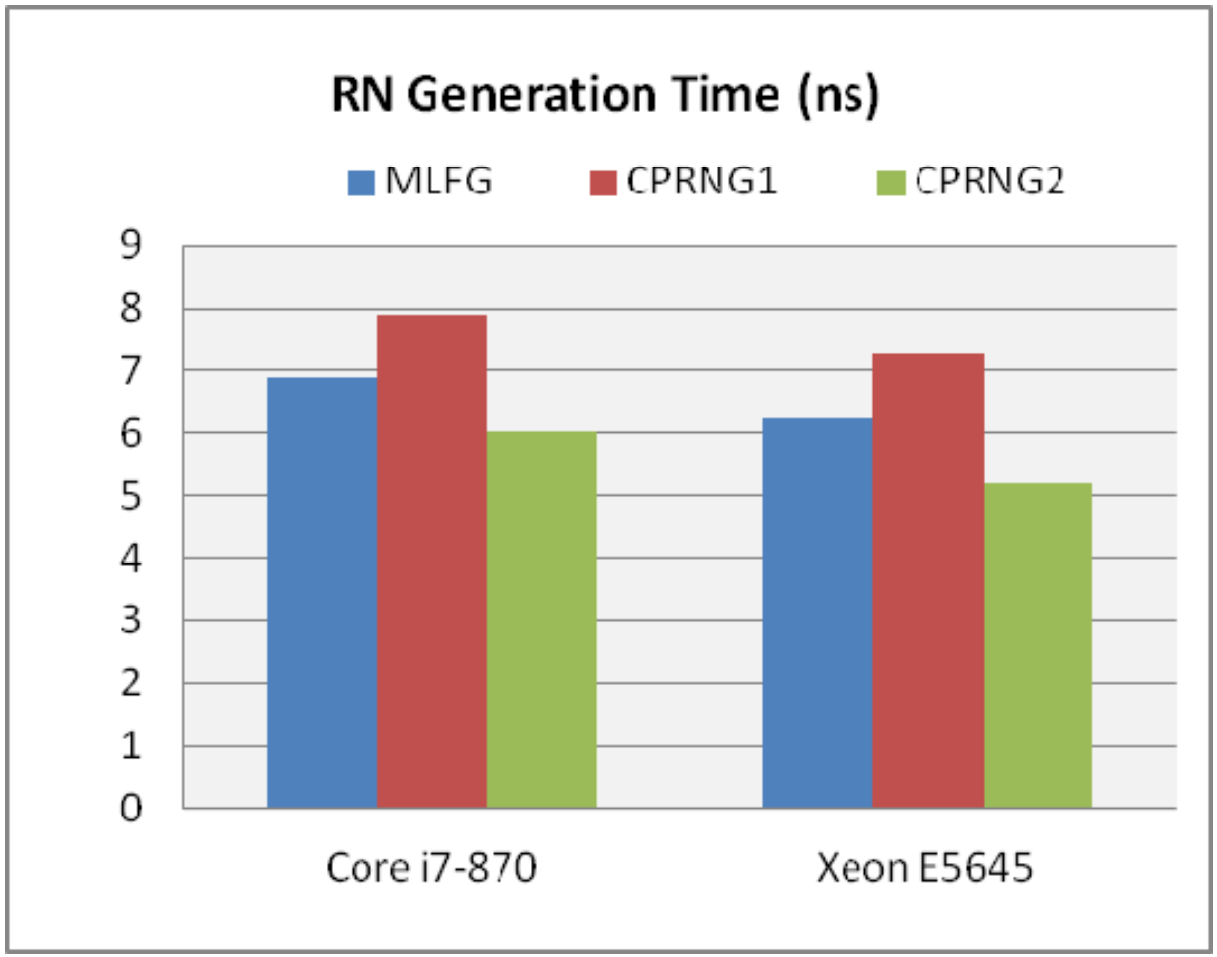


MLFG is part of SPRNG

CPRNG1 implemented in Phase I inside SPRNG package

CPRNG2 implemented in Phase II

Random Number Generation Time



CPRNG1 takes 1ns more than MLFG because of API compatibility issues

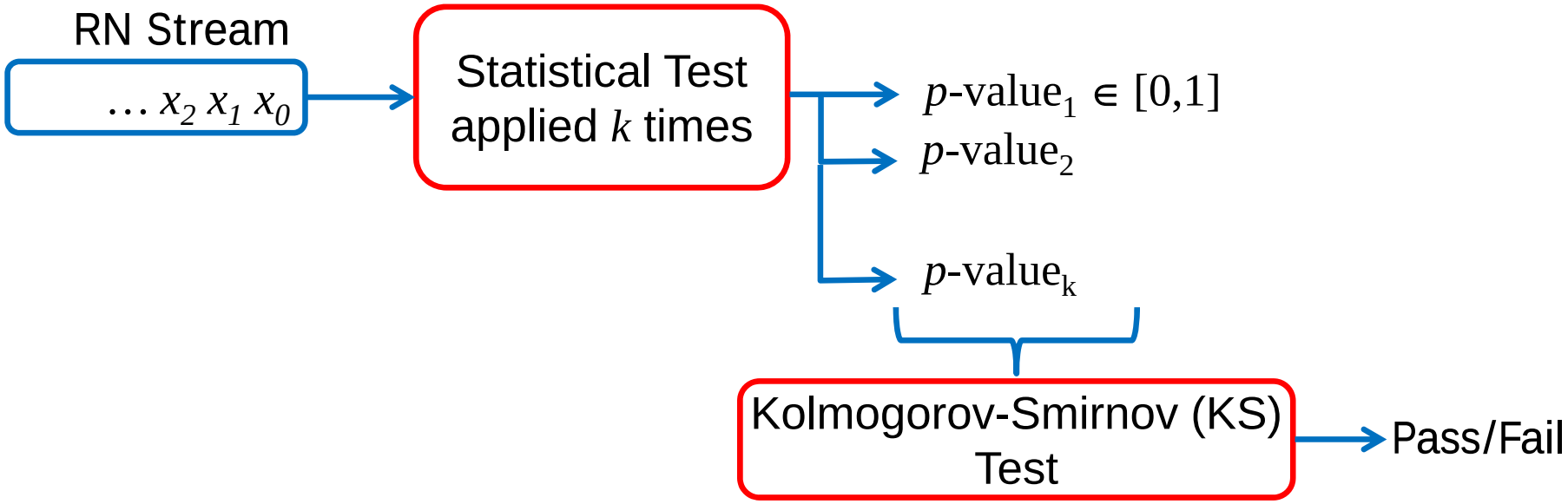
CPRNG2 does not have the same issues

Testing the randomness of parallel pseudorandom number generators

Single-stream Tests

Collisions
Coupon collector's
Equidistribution
Gap
Maximum-of-t
Permutations
Poker
Runs up
Serial
Random walk
Matrix rank test

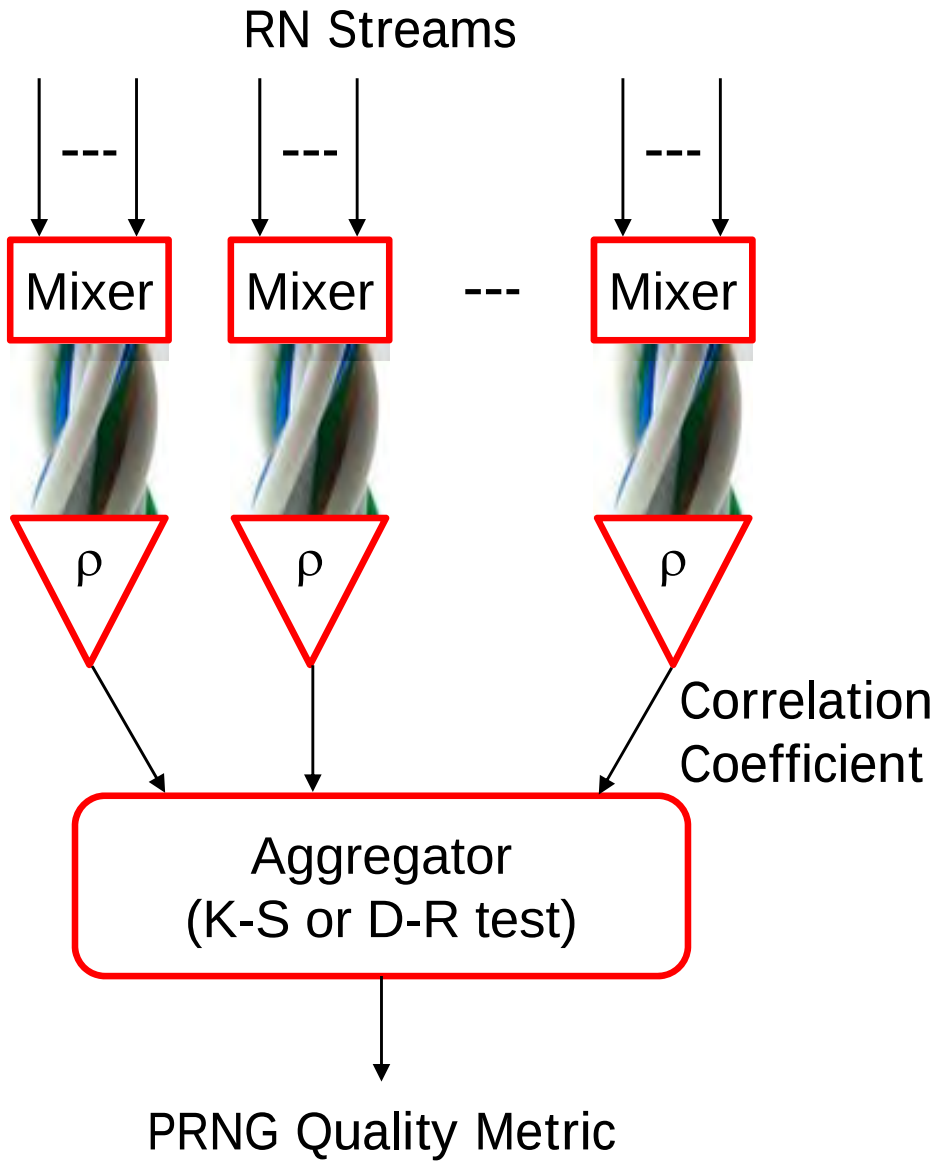
- Test packages with 15 to 150 single-stream tests
  - Dieharder, TestU01, SPRNG
- Empirical in nature
- Vector of pass/fail data: hard to compare different RNGs



Current Parallel RN-Stream Tests

- Single-stream tests evaluate intrastream correlations in a stream
- Interstream correlations of parallel RN streams are evaluated by
  - Interleaving the RN streams into a single stream
  - Applying single-stream tests
- Alternatively, simulations of system/process models with known exact solutions may be used
  - 2-D Ising model in Physics: Metropolis and Wolff algorithms

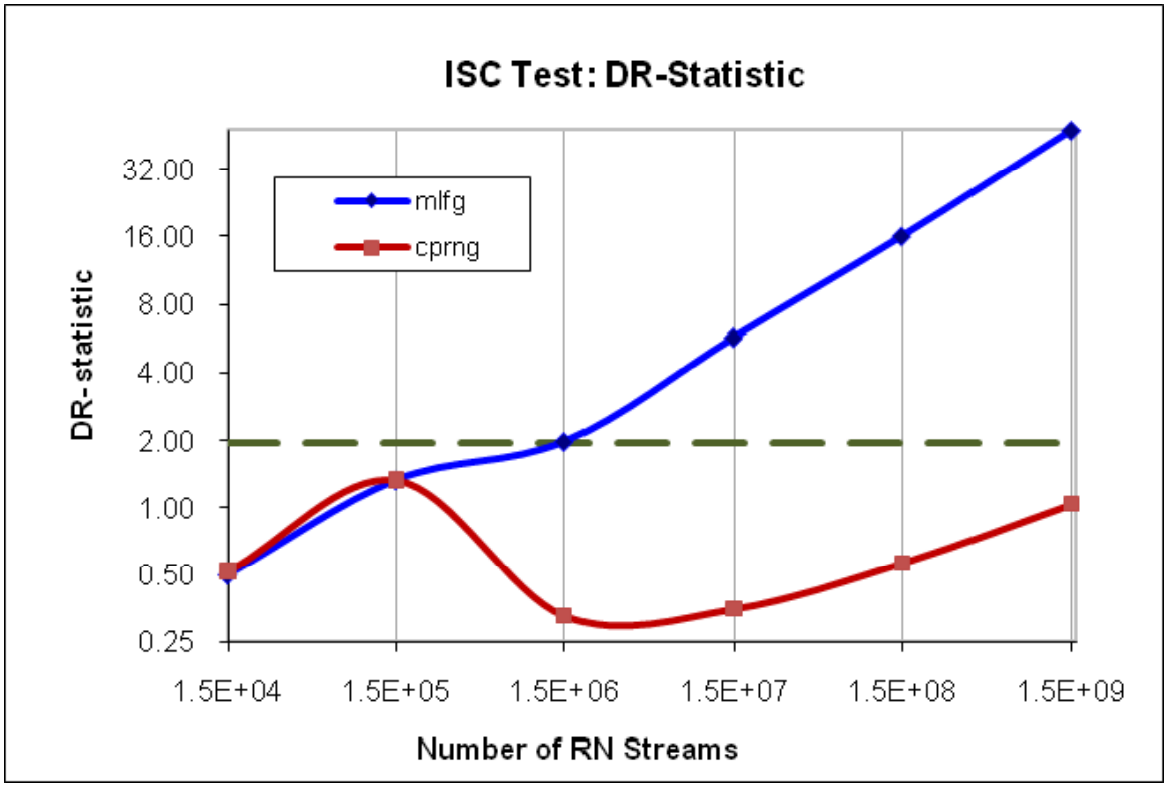
Interstream Correlation (ISC) Test



Application of ISC Test

- CPRNG and MLFG were tested
- Up to 1.5 billion RN streams used
  - 1500 sets of RN streams, 10 to 1 million per set
  - 100+ RNs from each stream
  - Shuffle interleaving of streams in a set
- Even the largest test case completes in 6 hours on a desktop with i7-870 CPU, 12 GB mem

ISC Test: Donner & Rosner Method

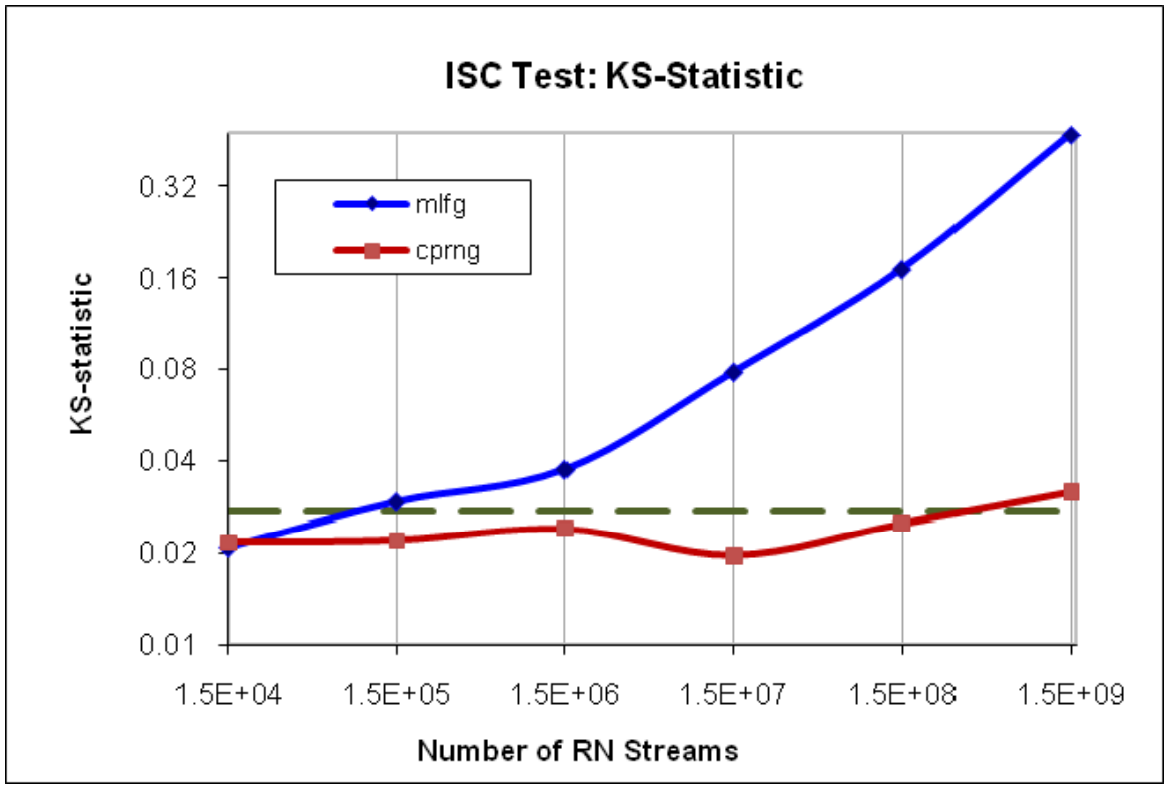


If the absolute value of the DR-statistic is above 1.96, then the correlations among the RN streams are considered significant. The probability of false conclusion is 0.05.

A set of 10 to 10^6 RN streams gives one cor. coeff., r

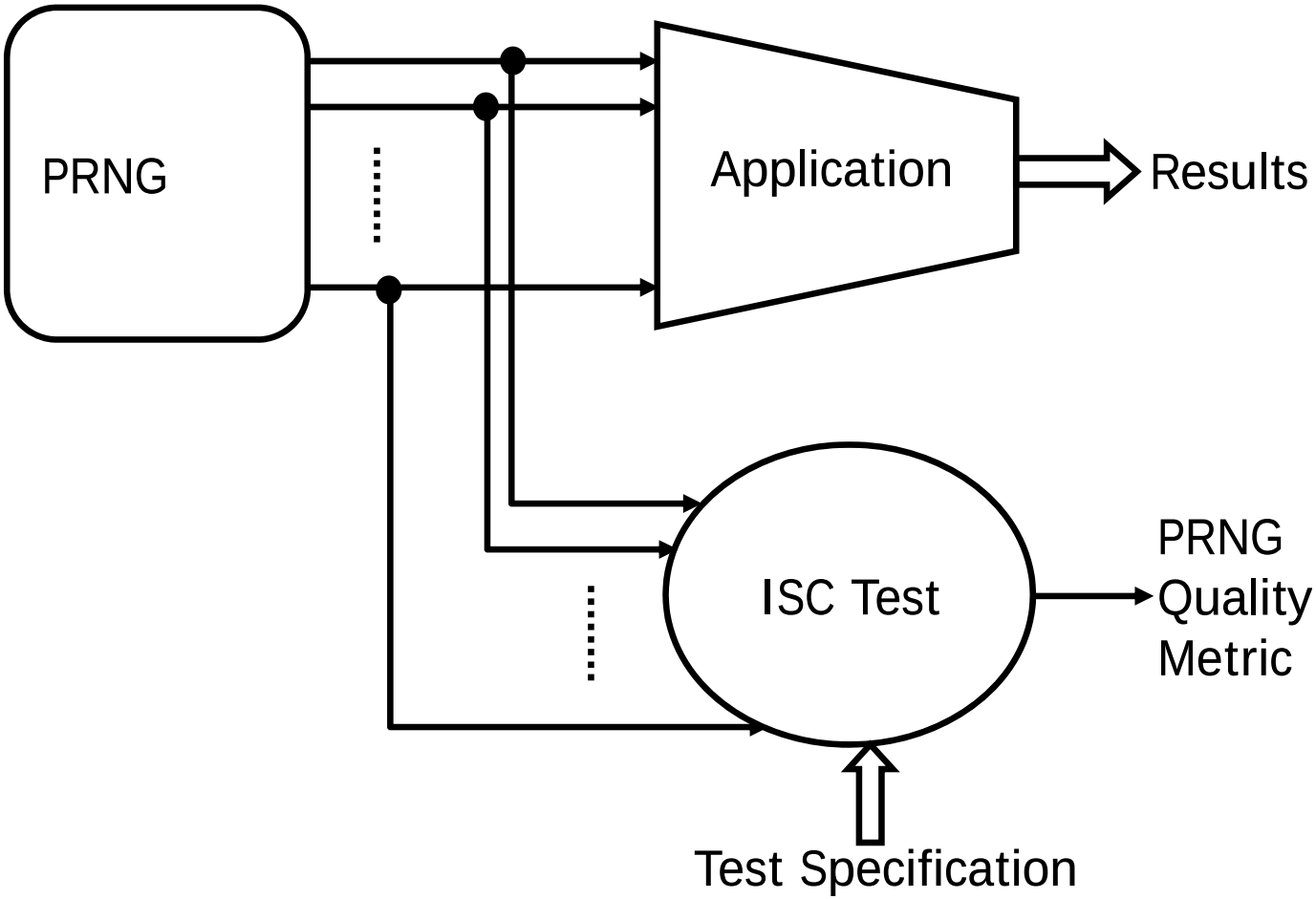
1500 r's are used to calculate the statistic

ISC Test: Kolmogorov-Smirnov Method



If the absolute value of the KS-statistic is above 0.0274, then the correlations among the RN streams are considered significant. The probability of false conclusion is 0.01.

Adapting ISC Test for Online Testing



Summary and Further Work

- CPRNG provides two new features that other generators do not provide
  - Distinct RN streams based on program context
  - Virtually unlimited number of RN streams without requiring the user to specify a maximum upper bound

- CPRNG is implemented with simple API
  - An API definition file and a library module
  - Wrappers can be built to adapt CPRNG for the applications that use SPRNG or default RNGs on various platforms
  - Seamless integration of CPU and GPU modes
    - The same streams can be used in both modes

Summary and Further Work

- ISC test: a new statistical test to evaluate interstream correlations of billions of RN streams
  - Can be adapted to provide a quality metric continually while the application runs and after it completes

- Ongoing work
  - Intel Xeon Phi implementation of CPRNG
  - Refine GPU implementation of CPRNG
  - Looking for HPC users
    - Use cases for CPRNG and ISC test
    - HPC application testing of ISC test and CPRNG context-awareness feature
    - Visit **`getcprng.com`** and register to obtain additional information

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